ESTIMATING ICEBERG DRAG COEFFICIENTS USING BAYESIAN INFERENCE

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August 8, 2018
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Motivation

Source: Soderman/NLSI Staff
Bayesian Inference Framework

Goal: Infer coefficients $\vec{\theta}$ given data $\vec{x}$

\[
\frac{d\vec{x}}{dt} = \vec{u}
\]

\[
m\frac{d\vec{u}}{dt} = \vec{F}(\theta)
\]

Prior: Reasonable assumptions based on past experience and knowledge.

Likelihood: A function describing the compatibility of the observed data with the model.

Posterior: Result of updating the prior given the new data.
Forward Model

\[ \vec{F}(\theta) = m\vec{a} \quad \vec{F}(\theta) = m\frac{d\vec{u}}{dt} \quad \vec{u} = \frac{d\vec{x}}{dt} \]

Damped Harmonic Oscillator:

\[ \vec{F}(\theta) = -\theta_1 \vec{F}_{\text{spring}} - \theta_2 \vec{F}_{\text{damping}} \]

Iceberg Model:

\[ \vec{F}(\theta) = \theta_1 \vec{F}_{\text{water}} + \theta_2 \vec{F}_{\text{air}} + \vec{F}_{\text{coriolis}} \]
Prior: A distribution allowing only non-negative values for $\theta_1$ and $\theta_2$.

Likelihood: A function showing model-data mismatch for each given $\theta$.

$$\pi(d|\theta) \propto \exp(\mathcal{L} (\|d - G(\theta)\|^2))$$
Bayes’ Rule

Posterior $\propto$ Prior $\cdot$ Likelihood

$$\pi(\theta|d) \propto \pi(\theta) \pi(d|\theta)$$

Goal: Generate samples from the posterior distribution
Method: Markov chain Monte Carlo (MCMC) sampling
Markov chain Monte Carlo (MCMC)

$\theta^{t+1}$ only depends on $\theta^t$

3 step algorithm (for $t = 1 \rightarrow \infty$):

1. Propose new point:
   $$\hat{\theta} \sim q(\cdot|\theta^t)$$

2. Compute acceptance rate $\alpha$:
   $$0 \leq \alpha(\theta^t, \hat{\theta}) \leq 1$$

3. Accept / Reject:
   $$\theta^{t+1} = \begin{cases} 
   \hat{\theta} & \text{with probability } \alpha \\
   \theta^t & \text{otherwise}
   \end{cases}$$
MCMC - Visualization

Source: The University of British Colombia, Ricky Chen
Recall the forward model:

\[
\frac{d\vec{x}}{dt} = \vec{u}
\]

\[
\vec{F}(\theta) = -\theta_1 \vec{F}_{\text{spring}} - \theta_2 \vec{F}_{\text{damping}}
\]
Data with Additive Noise
Sampled Posterior Distribution
Posterior Predictive Distribution
Recall the forward model:

\[
\frac{d\mathbf{x}}{dt} = \ddot{u}
\]

\[
m\frac{d\ddot{u}}{dt} = \theta_1 \mathbf{F}_{\text{water}} + \theta_2 \mathbf{F}_{\text{air}} + \mathbf{F}_{\text{coriolis}}
\]

\[
\mathbf{F}_{\text{air}}(x, y, t) = |\mathbf{v}_{\text{air}} - \mathbf{v}_{\text{ice}}|(\mathbf{v}_{\text{air}} - \mathbf{v}_{\text{ice}})
\]

\[
\mathbf{F}_{\text{water}}(x, y, t) = |\mathbf{v}_{\text{water}} - \mathbf{v}_{\text{ice}}|(\mathbf{v}_{\text{water}} - \mathbf{v}_{\text{ice}})
\]

Source: Mountain (1980)
Sample Forward Model Run
Approximate Prior v/s Posterior Predictive
Simplified Iceberg Model
Data with Additive Noise
Posterior Predictive Distribution
Conclusion
Questions
Damping Ratios of Oscillatory Systems

Source: Stuart Aitken, University of Leeds
Posterior Predictive Distribution