Volumes

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Definition of Volume

Let S be a solid that lies between $x = a$ and $x = b$. If the cross-sectional area of S in the plane P_x, through x and perpendicular to the x-axis, is $A(x)$, where A is a continuous function, then the **volume** of S is

$$V = \lim_{n\to\infty} \sum_{i=1}^{n} A(x_i^*)\Delta x = \int_a^b A(x)dx$$

Solids of revolution

A **solid of revolution** is a solid obtained by rotating (or revolving) an area around a line.

Cross-sectional areas of solids of revolution

The cross-sectional area $A(x)$ and $A(y)$ can be found in two ways:

- If the cross-section is a disk we use

$$A = \pi(\text{radius})^2$$

- If the cross-section is a washer we use

$$A = \pi(\text{outer radius})^2 - \pi(\text{inner radius})^2$$