Lines and Planes in $\mathbb{R}^3$

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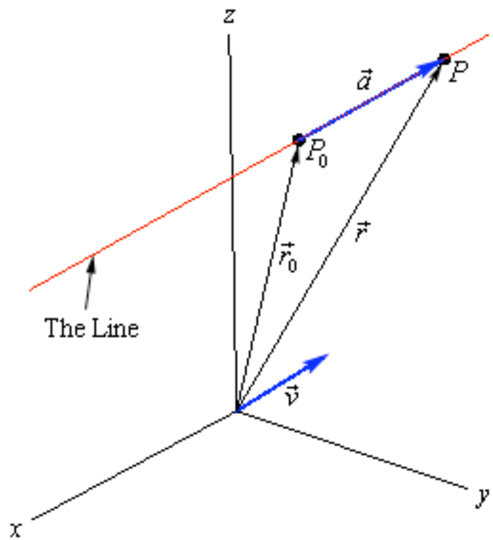
Lines in $\mathbb{R}^3$



Equations of a line in $\mathbb{R}^3$

Given a point $P(x_0, y_0, z_0)$ on the line and parallel vector $\mathbf{v} = \langle a, b, c \rangle$.

Vector Equation of the line: $\mathbf{r_0} = \langle x_0, y_0, z_0 \rangle$

$$\mathbf{r} = \mathbf{r_0} - t\mathbf{v}$$

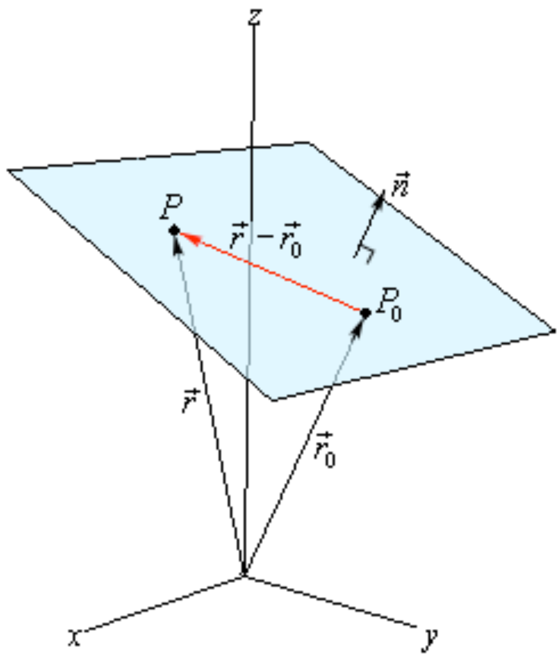
Parametric Equation:

$$x = x_0 + at$$
$$y = y_0 + bt$$
$$z = z_0 + ct$$

Symmetric Equation of a line:

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

A Plane in $\mathbb{R}^3$



Equation of a plane in $\mathbb{R}^3$

Given a vector normal (perpendicular) to the plane $\mathbf{n} = \langle a, b, c \rangle$ and a point (x_0, y_0, z_0) on the plane.
Let $\mathbf{r} - \mathbf{r_0} = \langle x - x_0, y - y_0, z - z_0 \rangle$

Vector Equation:

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r_0}) = 0$$

Scalar Equation:

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$