Absolute Convergence and Ratio Test

October 17, 2007

Absolute and Conditional Convergent Series

Definition: A series $\sum_{n=1}^{\infty} a_n$ is called **absolutely convergent** if the series of absolute values $\sum_{n=1}^{\infty} |a_n|$ is convergent.

Definition: A series $\sum_{n=1}^{\infty} a_n$ is called **conditionally convergent** if it is convergent but not absolutely convergent.

Absolute Convergence Implies Convergence

Theorem: If a series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, then it is convergent.

Ratio Test

(1) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent (therefore convergent).

(2) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L > 1$ or $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

(2) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$, then the ratio test is inconclusive.