Math 9 Fall 2003

Homework project II (20 points)

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Directional Derivatives and Differentiable Functions

Due date Monday, November 24, at the end of the lecture

Description of the Project

Let $f(x, y) : \mathbb{R}^2 \to \mathbb{R}$ be a function defined as follows:

(1)
$$f(x,y) = \begin{cases} 0 & \text{for } (x,y) = (0,0); \\ \frac{5xy^2}{x^2+y^2} & \text{for } (x,y) \neq (0,0). \end{cases}$$

(1) Prove that f defined this way is a continuous function on the whole real plane $\mathbb{R}^2$.
(2) Let $\mathbf{u} = \langle \frac{a}{\sqrt{a^2+b^2}}, \frac{b}{\sqrt{a^2+b^2}} \rangle$ be a unit vector. Find the value of the directional derivative $D_{\mathbf{u}} f(0,0)$ as a function of a and b.
(3) Find the gradient $\nabla f(0,0)$.
(4) Show that $D_{\mathbf{u}}(0,0) \neq \mathbf{u} \cdot \nabla f(0,0)$, for some vectors $\mathbf{u}$.
(5) Use Theorem 15.6.3 to conclude that the function $f(x,y)$ is not differentiable at the point $(0,0)$ despite the fact that it is continuous on $\mathbb{R}^2$ and has all the directional derivatives at $(0,0)$.

Hint: You can not conclude that f is continuous at $(0,0)$ as a ratio of two continuous functions, since the denominator is zero at $(0,0)$. For the same reasons you can not differentiate the function with respect to x and y variables at $(0,0)$ in the straightforward way. Instead use Definitions 15.2.1, 15.2.4, 15.3.4, 15.6.2, and 15.6.3 from the Stewart "Calculus" textbook.

REGULATIONS CONCERNING THIS PROJECT

1: Students are encouraged to work in groups of 3 or 4. **No group may consist of more than 4 students.** Students from different sections of Math 9 may work together in a single group.

2: Each group must submit **exactly one written report** to exactly one of the Math 9 instructors. Each student in a group is responsible for understanding everything in the written report and should be ready to present it to a professor if asked. **Write the names of all the students from the group and their instructors on the first page of the project.**

3: **The report must be written in claim-proof form and in full sentences.** (See "proof-writing suggestions" on the Math 9 homework assignments page.) Correct answers without proper justification are not acceptable.

4: Students will collaborate within one group, **but must not collaborate with members of other groups.**

5: Students are encouraged to consult mathematical textbooks and Math 9 instructors.

6: **The project is due on Monday November 24.** Each group must designate one member to submit the written report at **the beginning of that member's Math 9 lecture.**