HOMEWORK # 13, written assignment

If $\mathbf{v}$ is any vector in the plane, let $r_\theta(\mathbf{v})$ be the vector obtained form $\mathbf{v}$ by rotating it θ radians about its initial point. Briefy, r_θ is the function which rotates vectors by amount θ.

(a) If θ is not a multiple of 2π, then there is only one vector $\mathbf{v}$ satisfying $r_\theta(\mathbf{v}) = \mathbf{v}$. Which one?

(b) Vectors are drawn from the center of a regular n-sided polygon in the plane to the vertices of the polygon. Show that the sum of these vectors is $\mathbf{0}$, the zero vector. (*Hint:* Choose a suitable angle θ and use part (a) along with the fact, which you need not verify, that $r_\theta(\mathbf{v}+\mathbf{w}) = r_\theta(\mathbf{v}) + r_\theta(\mathbf{w})$ for any vectors $\mathbf{v}$ and $\mathbf{w}$ in the plane.)

(c) Assume now that the dimensions of the polygon are such that the vectors in part (b) are unit vectors. With this assumption, do part (b) another way by using complex numbers and the polynomial

$$x^n - 1 = (x-1)(x^{n-1} + x^{n-2} + \ldots + x + 1).$$

(*Hints:* What are the roots of this polynomial? If the polygon is suitably situated then the tips of the vectors in (b) correspond to the n n^{th} roots of the number 1.)