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Taylor Series

If we are clever, we can use Taylor series to evaluate the sums of certain series:

- $$\sum_{n=0}^{\infty} \frac{1}{2^n n!}$$
- $$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(2n+1)!}$$

Taylor Series Example

$$\boxed{f(x) = \ln(1 + x),\, a = e - 1}$$

- Expand around a different point: may be necessary if the initial radius ofcovergence is small.
- Can't find a pattern in the derivatives? Simply use the first few terms as an approximation.

Error estimates

Taylor's Inequality: If $|f^{(n+1)}(x)| \leq M$ for $|x-a| \leq d$ then

$$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$$

where

$$R_n(x) = \sum_{i=n+1}^{\infty} \frac{f^{(i)}(a)}{i!}(x-a)^i$$

$R_n(x)$ is called the *remainder* of the Taylor series and

$$f(x) = T_n(x) + R_n(x)$$

where $T_n(x)$ is the n^{th} degree Taylor polynomial of f.

Error Estimates

- Estimate $|\sin(x) - T_3(x)|$.

- How many terms of the Taylor series about 0 are required to calculate the value of $\sin(1)$ to within an error of $\frac{1}{10000}$?

AST Error Estimate

Alternating Series Test Estimate: Let $\sum_{i=0}^{\infty}(-1)^n a_n$ be an alternating series that converges to L. Then,

$$|s_m - L| \leq a_{m+1}$$

- How many terms of the Taylor series about 0 for $\arctan(x)$ are required to calculate the value of π to within an error of $\frac{1}{10000}$?