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Integral test estimates

- For most convergent series, we can not exactly compute the sum of the series.
- We can use the partial sums, s_m as estimates to the sum of the series.
- But,how good are these estimates? Using the integral test we can estimate the error:

> Suppose $f(k) = a_k$, where f is a continuous, positive, decreasing function for $x \geq n$ and $\sum_{i=n}^{\infty} a_i$ is convergent. If
>
> $$R_m = \sum_{i=n}^{\infty} a_i - \sum_{i=n}^{m} a_i$$
>
> then
>
> $$\int_{n+1}^{\infty} f(x)\,dx \leq R_n \leq \int_{n}^{\infty} f(x)\,dx$$

Integral test estimates

This also gives a succint estimate on the sum of the series:

Suppose $f(k) = a_k$, where f is a continuous, positive, decreasing function for $x \geq n$ and $\sum_{i=n}^{\infty} a_i$ is convergent. Then

$$s_n + \int_{n+1}^{\infty} f(x)\,dx \leq \sum_{i=n}^{\infty} a_i \leq s_n + \int_{n}^{\infty} f(x)\,dx$$

Comparison Test

Suppose $\sum_n a_n$ and $\sum_n b_n$ are series with positive terms.

1. If $\sum b_n$ converges and $a_n \leq b_n$ for all n then $\sum a_n$ converges.
2. If $\sum b_n$ diverges and $a_n \geq b_n$ for all n then $\sum a_n$ diverges.

Examples

- $$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$$
- $$\sum_{n=2}^{\infty} \frac{n}{n^2-n}$$
- $$\sum_{n=1}^{\infty} \frac{5}{2^n+3}$$
- $$\sum_{n=1}^{\infty} \frac{1+\sin(n)}{10^n}$$

Limit Comparison test

Suppose $\sum_n a_n$ and $\sum_n b_n$ are series with positive terms. If

$$\lim_{n\to\infty} \frac{a_n}{b_n} = L \neq 0$$

then both series converge or both series diverge.

Example:

$$\sum_{n=1}^{\infty} \frac{2n^2 + 3n}{\sqrt{5 + n^5}}$$