# Math 8: Calculus in one and several variables Spring 2018 - Homework 6 

Return date: Wednesday 05/09/18
keywords: derivatives along curves, functions of several variables, partial derivatives
Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification.
exercise 1. (3 points) Consider the curve given by

$$
\mathbf{r}(t)=\left\langle 3 t+2, \frac{1}{t}, t+1\right\rangle, \text { where } t \in \mathbb{R} \backslash\{0\} .
$$

a) Find all points on the curve for which the tangent vector is parallel to the plane $E: \frac{1}{3} x+\frac{1}{2} y+z=8$.
b) Find a parametrization of the tangent line of $\mathbf{r}(t)$ at $t=1$.
c) Find the unit tangent vector to $\mathbf{r}(t)$ at $t=1$.
exercise 2. (2 points) Find the arclength $\ell(c)$ of the curve

$$
c: \mathbf{r}(t)=\left\langle\frac{t^{2}}{2}, \frac{2 \sqrt{2}}{3} t^{3 / 2}, t\right\rangle \quad \text { where } 0 \leq t \leq 2
$$

exercise 3. (4 points) Find the domain of the following functions, then sketch several level curves and the graph of $f$ :
a) $f(x, y)=x^{2}-y+1$.
b) $f(x, y)=\frac{4}{x y}$.
exercise 4. (3 points) Consider the function

$$
f(x, y)=\frac{x^{2}+y^{2}}{2 x}
$$

Sketch several level curves $f(x, y)=k$, choosing at least two positive values of $k$ and at least two negative values of $k$.

Hint: After writing $f(x, y)=k$, clear the denominator, bring everything to one side of the equation and complete the square in $x$.

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exercise 5. (4 points) Compute the following limits or show that they do not exist.
a) $\lim _{(x, y) \rightarrow(0,0)} \frac{\cos \left(x^{4}+y^{4}\right)-1}{x^{4}+y^{4}}$.

Hint: Look at the Maclaurin series of $\cos (t)$.
b) $\lim _{(x, y) \rightarrow(0,0)} \frac{3 x y^{2}}{y^{4}+x^{2}}$.

Hint: Look at the curves $\mathbf{r}_{1}(t)=(t, 0)$ and $\mathbf{r}_{2}(t)=\left(t^{2}, t\right)$.
exercise 6. (4 points) Find the first and second order partial derivatives of

$$
f(x, y)=\exp \left(\frac{2 x}{y+1}\right)=e^{\frac{2 x}{y+1}}
$$

