Math 8: Calculus in one and several variables Spring 2018 - Homework 6

Return date: Wednesday 05/09/18

keywords: derivatives along curves, functions of several variables, partial derivatives

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification.

exercise 1. (3 points) Consider the curve given by

$$\mathbf{r}(t) = \left\langle 3t + 2, \frac{1}{t}, t + 1 \right\rangle$$
, where $t \in \mathbb{R} \setminus \{0\}$.

- a) Find all points on the curve for which the tangent vector is parallel to the plane $E: \frac{1}{3}x + \frac{1}{2}y + z = 8$.
- b) Find a parametrization of the tangent line of $\mathbf{r}(t)$ at t = 1.
- c) Find the unit tangent vector to $\mathbf{r}(t)$ at t = 1.

exercise 2. (2 points) Find the arclength $\ell(c)$ of the curve

$$c: \mathbf{r}(t) = \left\langle \frac{t^2}{2}, \frac{2\sqrt{2}}{3}t^{3/2}, t \right\rangle \quad \text{where} \quad 0 \le t \le 2$$

exercise 3. (4 points) Find the domain of the following functions, then sketch several level curves and the graph of f:

a) $f(x,y) = x^2 - y + 1$. b) $f(x,y) = \frac{4}{xy}$.

exercise 4. (3 points) Consider the function

$$f(x,y) = \frac{x^2 + y^2}{2x}.$$

Sketch several level curves f(x, y) = k, choosing at least two positive values of k and at least two negative values of k.

Hint: After writing f(x, y) = k, clear the denominator, bring everything to one side of the equation and complete the square in x.

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exercise 5. (4 points) Compute the following limits or show that they do not exist.

a)
$$\lim_{(x,y)\to(0,0)} \frac{\cos(x^4+y^4)-1}{x^4+y^4}$$

Hint: Look at the Maclaurin series of $\cos(t)$.

b) $\lim_{(x,y)\to(0,0)} \frac{3xy^2}{y^4 + x^2}$.

Hint: Look at the curves $\mathbf{r}_1(t) = (t, 0)$ and $\mathbf{r}_2(t) = (t^2, t)$.

exercise 6. (4 points) Find the first and second order partial derivatives of

$$f(x,y) = \exp\left(\frac{2x}{y+1}\right) = e^{\frac{2x}{y+1}}.$$