Math 8

Minima and Maxima on Surfaces

Practice Problems

1) Find the local minima, maxima, and saddle points for the following functions:

a) $f(x, y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$.

b) $f(x, y) = \sin x \sin y$, where $-\pi \leq x \leq \pi$ and $-\pi \leq y \leq \pi$.

c) $f(x, y) = e^x \cos y$.

2) Find the rectangular box with the largest volume such that it lies in the first octant, three of its faces touch the 3 coordinate planes, and one of its vertices lies on the plane: $x + 2y + 3z = 6$.

3) Find the points on the surface $z^2 = x^2 + y^2$ that are closest to the point $(4, 2, 0)$. (Hint: Consider the distance squared.)

4) A box is designed so that its surface area is exactly 10 cm^2. What dimensions would maximize the boxes volume?

5) Consider the plane $x - 2y + 3z = 6$:

a) Find the point A on the plane that is closest to the point $B = (0, 1, 0)$. (Note: We had a problem like this on HW #4. In this problem use derivatives to find this point A.)

b) What is the normal vector to our plane?

c) How is the vector $\vec{AB}$ related to the normal vector?

6) Find the absolute maximum and minimum values of the function

$$f(x, y) = 4x + 6y - x^2 - y^2,$$

where $0 \leq x \leq 4$ and $0 \leq y \leq 5$.

7) Find the absolute maximum and minimum values that the function $f(x, y) = xe^{-x^2-y^2}$ takes when $x^2 + y^2 \leq 4$.