Math 8
Directional Derivatives

Practice Problems

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1) Find the directional derivative of the function at the given point and in the indicated direction $\vec{v}$:

a) $f(x,y) = xe^y - ye^x$ at $(1,0)$, with $\vec{v} =< 3,4 >$.

b) $f(x,y) = \frac{2x}{x-y}$ at $(1,3)$, with $\vec{v} =< 1, \sqrt{3} >$.

b) $f(x,y) = y\cos(x) + x\sin(y)$ at $(0, 3\pi/2)$, with $\vec{v} =< 7, -2 >$.

2) Find the directional derivative of $f(x,y) = (x-1)ye^{xy}$ at the point $(1,4)$ and in the direction of the point $(2,5)$.

3) Find the unit vector $\vec{u}$ that maximizes the directional derivative $f_{\vec{u}}$ at the given point. What is the the rate of change in this direction?

a) $f(x,y) = x^2 + 4y^2$ at $(0,1)$

b) $f(x,y) = \dfrac{y^2}{(x+1)}$ at $(3,1)$

4) Find all directions $\vec{u} =< a, b >$ such that $f_{\vec{u}}(0,2) = 1$ where $f(x,y) = ye^{-xy}$.

5) Let f be a function of two variables and consider the points $A = (1,3)$, $B = (3,3)$, $C = (1,7)$, and $D = (6,15)$. If

$$f_{\vec{AB}}(1,3) = 3 \qquad \text{and} \qquad f_{\vec{AC}}(1,3) = 26,$$

then what is $f_{\vec{AD}}(1,3)$?

6) Suppose your are climbing up a hill whose shape is given by the equation

$$z = 1000 - \frac{x^2}{100} - \frac{y^2}{50},$$

so that the positive x-axis points east and the positive y-axis points north. If your current (x,y) location is $(10,6)$ then...

a) If you walk due south will you start to ascent or descend? At what rate?

b) If you walk northwest, will you start to ascent or descent? At what rate?

c) In which direction (as a vector) is the slope the largest? What is the rate of ascent in this direction?

Problem to Turn In

1) Assume you are walking along the surface $f(x,y) = e^{x^2+y^2}$ so that your (x,y) location as a function of time is given by $r(t) =< 1+t^2, t^4 >$. Compute the directional derivative when $t = 1$. (Hint: What direction in the (x,y)-plane are you walking when $t = 1$.)

2) Intuitively, if you are out hiking there is always some direction you can walk so that your elevation remains constant, i.e., you are neither walking uphill nor downhill. Verify this mathematically by showing that there is always some direction $\vec{u}$ so that $f_{\vec{u}}(x,y) = 0$. What is the angle between $\vec{u}$ and $\nabla f(x,y)$?