Math 8

Chain Rule

Practice Problems

1) Using the chain rule find $\dfrac{dz}{dt}$ for the following:

   a) $z = x^2 + y^5 + 2x^2y, \ x = \ln t, \ y = t^2 + t + 1$

   b) $z = x^2y^2, \ x = \cos t, \ y = \sin t$

2) Using the chain rule find the following:

   a) For $z = \sqrt{x + y}, \ x = st + s + t, \ y = s\sqrt{t}$;
   find $\dfrac{dz}{dt}$ and $\dfrac{dz}{ds}$ when $(s, t) = (1, 2)$.

   b) $z = x^2 + xy + y^2, \ x = r\cos\theta, \ y = r\sin\theta$;
   find $\dfrac{dz}{dr}$ and $\dfrac{dz}{d\theta}$ when $(r, \theta) = (3, \pi/2)$.

3) The radius of a right circular cone is increasing at a rate of 2cm/sec while its height is decreasing at a rate of 4cm/sec. At what rate is the volume of the cone changing when the radius is 10cm and the height is 7cm.
(Recall: the volume of a right circular cone is $\frac{\pi r^2 h}{3}$.)

Problem to Turn In

1) Let $z = \dfrac{x + y}{x + 1}, \ x = ue^p, \ y = pe^u$. Using the chain rule find $\dfrac{\partial}{\partial u}\left(\dfrac{\partial z}{\partial p}\right)$
when $(u, p) = (1, -3)$.

2) The temperature at a point (x, y) is given by the function $T(x, y)$. Assume a bug crawls along the (x, y)-plane so that its position, at each point of time, is given by

$$x(t) = \sqrt{1 + t} \qquad y(t) = 1 + 2t.$$

a) Draw the path taken by the bug (assume $0 \le t$).
b) Find dT/dt when $t = 3$, assuming $T_x(2, 7) = -5$ and $T_y(2, 7) = 10$. In a few words, describe the meaning of the quantity dT/dt in this context.