Math 8
Homework Set #6
Alternating Series

Practice Problems

Determine if the following series converge or diverge:

1) $\frac{1}{4} - \frac{1}{6} + \frac{1}{8} - \frac{1}{10} + \cdots$

2) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt[4]{n}}$

3) $\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{5+n^4}$

4) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n \ln n}$

5) $\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n-1}$

6) $\sum_{n=1}^{\infty} (-1)^n \left(1 - \cos\left(\frac{\pi}{n}\right)\right)$

7) $\sum_{n=1}^{\infty} \left(-\frac{5}{n}\right)^n$

8) What is the smallest value of k needed to guarantee that the approximation $\sum_{n=1}^{k} (-1)^n \frac{1}{n^2}$ is within 1/1000 of the exact value $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2}$?

Problems to Turn In

On the first day of class we showed that

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \cdots$$

The right hand side raised a big theoretical question at the time: is it possible to add up an infinite number of numbers and not get infinity?

Well, now we have the tools to answer this question and moreover to make sense of Taylor Series (i.e., "infinite polynomials")! Observing that the Taylor Series for $\sin x$ is an **alternating series**, answer the following questions:

a) Find all the values of x that make this series converge.

b) When $x = 2$ what is the smallest value of k needed so that adding up the first k terms of this series is within 1/1000 of the actual value, i.e., $\sin(2)$?

Note: This is exactly how your calculator computes $\sin(2)$.