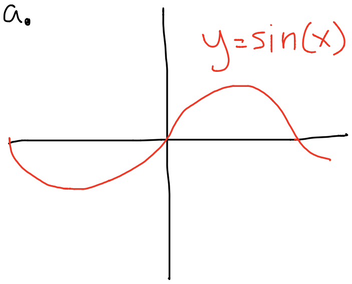
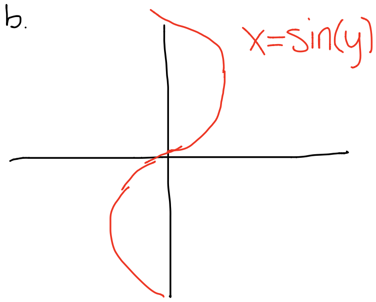
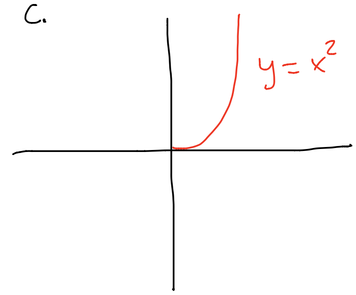
1) a.



b.



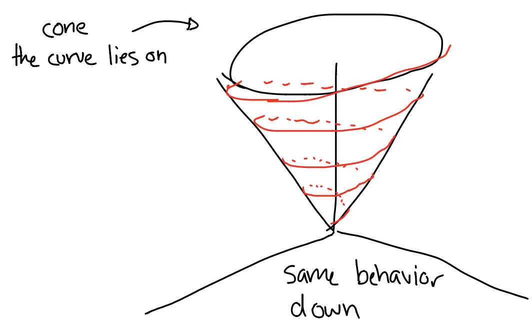
c.



t, -t go to same value

2) Note at time t, (x,y) lies on a circle of radius t.
Therefore for $0<t$ $\langle t\cos(t), t\sin(t), t\rangle$ will spiral upward and outward, an everexpanding helix starting at the origin. For $t<0$, we see the opposite spiral traveling downward.

3) The intersection of $x^2+y^2=4$
and $z=xy$



Recall a circle of radius 2 has parameterization $\langle 2\cos(t), 2\sin(t)\rangle$
so the intersection has parameterization $\langle 2\cos(t), 2\sin(t), 4\cos(t)\sin(t)\rangle$

4) 21) II 22) IV 23) V

24) I 25) IV 26) III

5) $\vec{r}(t)=\langle 3t, 0, 2t-t^2\rangle$ intersects $z=x^2+y^2$ when

$$2t-t^2=(3t)^2 \iff 2t-10t^2=0 \text{ so } t=0 \text{ or } t=1/5$$

6) a. $\vec{r}(t)=\langle \cos t, \sin t, t\rangle$ at $t=2\pi$

$$\vec{r}\,'(t)=\langle -\sin(t), \cos(t), 1\rangle \text{ so } \vec{r}\,'(2\pi)=\langle 0,1,1\rangle$$

The tangent line has equation $\langle 1, t, 2\pi+t\rangle$

b. $\vec{r}(t)=\langle e^t, e^{-t}, -\ln(t)\rangle$ at $t=1$

$$\vec{r}\,'(t)=\langle e^t, -e^{-t}, -\tfrac{1}{t}\rangle \text{ so } \vec{r}\,'(1)=\langle e, -\tfrac{1}{e}, -1\rangle$$

The tangent line has equation $\langle e+et, \frac{1}{e}-\frac{t}{e}, -t\rangle$

7) $\vec{r}(t) = \langle t, 1+t^2 \rangle$ so $\vec{r}\,'(t) = \langle 1, 2t \rangle$

a) $\vec{r}(t)$ and $\vec{r}\,'(t)$ are orthogonal when

$\vec{r}(t)\cdot\vec{r}\,'(t) = 0$, so $t + 2t + 2t^3 = 0 \iff t(\overset{\neq 0 \text{ for all } t}{3+2t^2}) = 0$ so $t = 0$

b and c) To point in the same direction, we need

$\vec{r}(t) = \lambda \vec{r}\,'(t)$ for some constant λ.

Then $\langle t, 1+t^2 \rangle = \langle \lambda, \lambda 2t \rangle$ so $\lambda = t$, $1+t^2 = 2t^2 \Rightarrow t^2 = 1$.

For $t=1$, they point in the same direction. For $t=-1$, they point in opposite directions

8) $\vec{r}(t) = \langle 3\cos^3 t, 3\sin^3 t, 6 \rangle$ so $\vec{r}\,'(t) = \langle -9\sin(t)\cos^2 t, 9\sin^2 t\cos t, 0$

Then $|\vec{r}\,'(t)| = \sqrt{81\sin^2 t\cos^4 t + 81\sin^4 t\cos^2 t} = \sqrt{81\sin^2 t\cos^2 t(\cos^2 t + \sin^2 t)}$

$= 9|\sin t\cos t|$

The arclength from 0 to 2π is $9\int_0^{2\pi} |\sin t\cos t|\,dt = 9\cdot 4\int_0^{\pi/2} \sin(t)\cos(t)\,dt$

$|\sin(t)\cos(t)|$ behaves the same for $[0, \pi/2]$, $[\frac{\pi}{2}, \pi]$, $[\pi, \frac{3\pi}{2}]$, $[\frac{3\pi}{2}, 2\pi]$

$= 18\left[\sin^2 t\right]_0^{\pi/2} = 18$

9) $\vec{r}(t) = \langle \frac{t^2}{4} - \frac{\ln(t)}{2}, -1, t \rangle$ so $\vec{r}\,'(t) = \langle \frac{t}{2} - \frac{1}{2t}, 0, 1 \rangle$

Then $|\vec{r}\,'(t)| = \sqrt{(\frac{t}{2} - \frac{1}{2t})^2 + 1} = \sqrt{\frac{t^2}{4} - \frac{1}{2} + \frac{1}{4t^2} + 1} = \sqrt{\frac{t^2}{4} + \frac{1}{2} + \frac{1}{4t}} = \frac{t}{2} + \frac{1}{2t}$

so the arclength from 1 to 2 is $\int_1^2 (\frac{t}{2} + \frac{1}{2t})\,dt = \left[\frac{t^2}{4} + \ln(t)\right]_1^2 = 1 + \ln(2) - \frac{1}{4}$

$= \frac{3}{4} + \ln(2)$