Math 8
Homework Set #5
Integral & Comparison Test

Practice Problems

Determine if the following series converge or diverge:

1) $\dfrac{2}{5}+\dfrac{2}{8}+\dfrac{2}{11}+\dfrac{2}{14}+\cdots$

2) $\displaystyle\sum_{n=1}^{\infty}\frac{n^2}{\sqrt{1+n^4}}$

3) $\displaystyle\sum_{n=2}^{\infty}\frac{5}{n(\ln n)^2}$

4) $\displaystyle\sum_{n=2}^{\infty}\frac{n}{n^4-1}$

5) $\displaystyle\sum_{n=1}^{\infty}\frac{\cos^2 n}{n^2}$

6) $\displaystyle\sum_{n=1}^{\infty}\frac{\ln n}{n^3}$

7) $\displaystyle\sum_{n=1}^{\infty}\frac{\sin\left(\frac{1}{n}\right)}{n^2}$

As we mentioned in class it is often extremely difficult, if not impossible, to find the exact value of a series. To get around this problem it is common practice to approximate the exact value of a series by adding up the first 10, 100, 1000, or more terms of the series. For example, if our series is $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}$ then adding up the first 10, 100, or 1000 terms yields the following approximations:

$$\sum_{n=1}^{10}\frac{1}{n^2}=1.5497 \qquad \sum_{n=1}^{100}\frac{1}{n^2}=1.63498 \qquad \sum_{n=1}^{1000}\frac{1}{n^2}=1.6439$$

The natural question is how many terms do we have to add up to insure that our approximation is good? To answer this observe that the **error** between the exact value of the series and our approximation (with 100 terms) of the series is

$$\underbrace{\sum_{n=1}^{\infty}\frac{1}{n^2}}_{\text{exact value}} - \underbrace{\sum_{n=1}^{100}\frac{1}{n^2}}_{\text{approx.}} = \underbrace{\sum_{n=101}^{\infty}\frac{1}{n^2}}_{\text{error}}$$

To answer this question we need to add up enough terms so that our error term is very small. In fact, we can say this about the error term:

$$\sum_{n=k+1}^{\infty}\frac{1}{n^2} \leq \int_k^{\infty}\frac{1}{x^2}dx. \tag{1}$$

Using this fact about the error term (you do **not** need to prove it) you can now answer the initial question!

8) What is the smallest value of k needed to guarantee that the approximation $\displaystyle\sum_{n=1}^{k} \frac{1}{n^2}$ is within 1/1000 of the exact value $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$?

Optional Problem: Explain why the inequality given in (1) is valid.

Problems to Turn In

1) Confirm, in two different ways, that the series $\displaystyle\sum_{n=1}^{\infty} ne^{-n}$ converges. First using the ratio test and second using the integral test.

2) Explain why it is true that if $\displaystyle\sum_{n=1}^{\infty} a_n$ converges and $0 < a_n$ then $\displaystyle\sum_{n=1}^{\infty} a_n^2$ also converges.