1) Let $\vec{v} = \langle 1,1,1 \rangle$. Note $|\vec{v}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$, so $\frac{\vec{v}}{\sqrt{3}} = \left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$ has the same direction as $\vec{v}$ with $\left|\frac{\vec{v}}{\sqrt{3}}\right| = \frac{\sqrt{3}}{\sqrt{3}} = 1$.

Then $3 \cdot \frac{\vec{v}}{\sqrt{3}} = \sqrt{3}\vec{v} = \langle \sqrt{3}, \sqrt{3}, \sqrt{3} \rangle$ has length 3.

2) $\vec{a} = \langle 1,-3,4 \rangle$, $\vec{b} = \langle 0,3,7 \rangle$, $\vec{c} = \langle 1,2,3 \rangle$

a) $\vec{a}\cdot\vec{b} = 1\cdot 0 + (-3)3 + 4\cdot 7 = -9+28 = 19$

b) $\vec{b}\times\vec{c} = i(3\cdot 3 - 2\cdot 7) - j(0\cdot 3 - 1\cdot 7) + k(0\cdot 2 - 1\cdot 3)$

$$\begin{matrix} i & j & k \\ 0 & 3 & 7 \\ 1 & 2 & 3 \end{matrix} \quad = -5i + 7j - 3k$$

c) $\vec{a}\times(\vec{b}\times\vec{c}) = \vec{a}\times\langle -5,7,-3 \rangle$

$$\begin{matrix} i & j & k \\ 1 & 3 & -4 \\ -5 & 7 & -3 \end{matrix} \quad = i(3(-3)-(-4)7) - j(1(-3)-(-5)(-4)) + k(1\cdot 7 - (-5)3)$$

$= 19i + 23j + 22k$

d) $\vec{a}\cdot(\vec{a}\times\vec{b}) = 1\cdot 33 + 3(-7) + (-4)\cdot 3$
$= 0$

Can see this directly as $\vec{a}$ and $\vec{a}\times\vec{b}$ are perpendicular, so $\vec{a}\cdot(\vec{a}\times\vec{b})$ must be 0

$\vec{a}\times\vec{b} = i(3\cdot 7 - (-4)3) - j(1\cdot 7 - 0\cdot 4) + k(1\cdot 3 - 0\cdot 3)$

$$\begin{matrix} i & j & k \\ 1 & 3 & -4 \\ 0 & 3 & 7 \end{matrix} \quad = 33i - 7j + 3k$$

3) $\vec{a} = \langle 2,4,6 \rangle$, $\vec{b} = \langle -1,8,0 \rangle$ $\quad \vec{a}\cdot\vec{b} = 2(-1)+4\cdot 8+6\cdot(0) = 30$ $\quad |\vec{a}| = \sqrt{2^2+4^2+6^2} = \sqrt{56}$

$|\vec{b}| = \sqrt{(-1)^2+8^2+0^2} = \sqrt{65}$

Recall $\cos(\theta) = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{30}{\sqrt{56}\sqrt{65}}$ where θ is the angle between $\vec{a}$ and $\vec{b}$, so $\theta = \cos^{-1}\left(\frac{30}{\sqrt{56}\sqrt{65}}\right) \approx 1.05$ radians
60.18°

4) The diagonal of a unit cube is $\langle 1,1,1 \rangle$. One edge of the cube is $\langle 1,0,0 \rangle$, so

$\cos(\theta) = \frac{1\cdot 1 + 1\cdot 0 + 1\cdot 0}{\sqrt{3}\cdot 1} = \frac{1}{\sqrt{3}}$. Then $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx .955$ radians
54.7°

5) $\overset{\vec{a}}{\langle -6,b,2 \rangle}$ and $\overset{\vec{b}}{\langle b,b^2,b \rangle}$ are orthogonal if $\vec{a}\cdot\vec{b} = 0$ with $\vec{a}\neq 0, \vec{b}\neq 0$

so $\quad -6b + b^3 + 2b = 0 \quad$ or $\quad b^2 - 4 = 0$

so when $b=2$ or $b=-2$

6) $\vec{a} = \langle 1,-2,3 \rangle$ $\quad \vec{b} = \langle -3,2,-1 \rangle$ $\quad \vec{a}\times\vec{b}$ will be perpendicular to both

$$\vec{a}\times\vec{b} = i\big((-2)(-1)-(-2)3\big) - j\big(1(-1)-(-3)3\big) + k\big(1(-2)-(-3)(-2)\big)$$

$$\begin{array}{ccc} i & j & k \\ 1 & -2 & 3 \\ -3 & -2 & -1 \end{array} \qquad = 8i - 8j - 8k = 8\langle 1,-1,-1 \rangle$$

Note $\vec{a}\cdot(\vec{a}\times\vec{b}) = 8\big(1\cdot1+(-2)(-1)+3(-1)\big) = 0$
and $\vec{b}\cdot(\vec{a}\times\vec{b}) = 8\big((-3)1+(-2)(-1)+(-1)(-1)\big) = 0$ $\Big\}$ check answer

7) $P=(0,-2,0)$, $Q=(4,1,2)$, $R=(5,3,1)$ — The triangle defined by P, Q and R is half of the parllelogram defined by $\vec{PQ}$ and $\vec{PR}$.

Let $\vec{PQ} = \langle 4-0, 1-(-2), 2-0 \rangle = \langle 4,3,2 \rangle$
$\quad\;\; \vec{PR} = \langle 5-0, 3-(-2), 1-0 \rangle = \langle 5,5,1 \rangle$

The area of the triangle is $\frac{1}{2}\left|\vec{PQ}\times\vec{PR}\right| = \frac{1}{2}\sqrt{110}$

$$\vec{PQ}\times\vec{PR} = i(3\cdot1-5\cdot2) - j(4\cdot1-5\cdot2) + k(4\cdot5-5\cdot3)$$

$$\begin{array}{ccc} i & j & k \\ 4 & 3 & 2 \\ 5 & 5 & 1 \end{array} \qquad = -7i + 6j - 5k$$

$$\left|\vec{PQ}\times\vec{PR}\right| = \sqrt{(-7)^2+6^2+(-5)^2} = \sqrt{110}$$

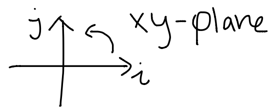
8) If $\vec{a}, \vec{b}$ are parallel, then $\vec{b} = c\vec{a}$ for some constant c, so

$$\vec{a}\times\vec{b} = \vec{a}\times(c\vec{a}) = c(\vec{a}\times\vec{a}) = c\big(i(a_2a_3-a_3a_2) - j(a_1a_3-a_3a_1) + k(a_1a_2-a_2a_1)\big)$$

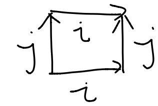
$$\begin{array}{ccc} i & j & k \\ a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 \end{array} \qquad = 0$$

9) $\vec{a}\cdot(\vec{a}\times\vec{b}) = 0$ for any vectors $\vec{a}$ and $\vec{b}$ as $\vec{a}\times\vec{b}$ is perpendicular to $\vec{a}$ and the dot product of perpendicular vectors is 0

10) a) $i \times j = k$



Righthand rule says $i\times j$ points in k-direction, magnitude is the area of



, or $\underline{1}$.

similarly,

b) $j \times k = i$ $\qquad$ c) $i \times k = -j$

d) $(i\times j)\times j = k \times j = -j\times k = -i$.