Math 8

Dot Product & Cross Product

Practice Problems

1) Find vectors in the same direction as $< 1, 1, 1 >$ that have length 1 and 3.

2) Compute the following where $\vec{a} =< 1, -3, 4 >$ and $\vec{b} =< 0, 3, 7 >$ and $\vec{c} =< 1, 2, 3 >$.

a) $\vec{a} \cdot \vec{b}$

b) $\vec{b} \times \vec{c}$

c) $\vec{a} \times (\vec{b} \times \vec{c})$

d) $\vec{a} \cdot (\vec{a} \times \vec{b})$

3) Find the angle between the vectors $\vec{a} =< 2, 4, 6 >$ and $\vec{b} =< -1, 8, 0 >$.

4) Find the angle between the diagonal of a cube and one of its edges.

5) For what values of b are the vectors $< -6, b, 2 >$ and $< b, b^2, b >$ orthogonal?

6) Find a vector orthogonal to $\vec{a} =< 1, -2, 3 >$ and $\vec{b} =< -3, 2, -1 >$. Check your answer using the dot product.

7) Find the area of the triangle determined by the points $(0, -2, 0)$, $(4, 1, 2)$, and $(5, 3, 1)$.

Note: the following three problems can be answered without doing any calculations. Instead, appeal to the meaning of the dot product and cross product.

8) Assume $\vec{a}$ and $\vec{b}$ are parallel. Explain why $\vec{a} \times \vec{b} =< 0, 0, 0 >$?

9) Explain why $\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$ must always hold for all vectors $\vec{a}$ and $\vec{b}$.

10) Determine the following using the right hand rule.

a) $i \times j$

b) $j \times k$

b) $i \times k$

c) $(i \times j) \times j$

Recall: $i =< 1, 0, 0 >, j =< 0, 1, 0 >$ and $k =< 0, 0, 1 >$.

Problems to Turn In

1) Find the angle between a diagonal of a cube and a diagonal of one of its **faces**.

2) Assume $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} =< 0, 0, 0 >$. Explain why $\vec{a} =< 0, 0, 0 >$ or $\vec{b} =< 0, 0, 0 >$. No calculation is necessary!

3) Let $\vec{a} =< -1, 3, 0 >$ and $\vec{b} =< -1, 3, 6 >$.

a) Find the scalar projection ℓ of $\vec{b}$ onto $\vec{a}$.

b) Find the vector $\vec{c}$ in the direction of $\vec{a}$ with length ℓ.

c) Show that $(\vec{b} - \vec{c}) \cdot \vec{a} = 0$. Explain why this must be the case for any vectors $\vec{a}$ and $\vec{b}$.