Math 8
Homework Set #3

Ratio & Root Tests

Determine whether each of the following series converge or diverge and explain your reasoning.

1) $\displaystyle\sum_{n=1}^{\infty} \frac{n^7}{7^n}$

2) $\displaystyle\sum_{n=1}^{\infty} \arctan(e^{-n})$

3) $\displaystyle\sum_{n=1}^{\infty} \frac{3^{2n+1}}{(2n+1)!}$

4) $1 + \dfrac{1 \cdot 3}{3!} + \dfrac{1 \cdot 3 \cdot 5}{5!} + \dfrac{1 \cdot 3 \cdot 5 \cdot 7}{7!} + \cdots$

5) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n}$

6) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n^2+3}{3n^2+2}\right)^{5n}$

7) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

8) $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{1}{2n}\right)^{n^2}$

9) $\displaystyle\sum_{n=1}^{\infty} \left(\sqrt{4n+1}\sqrt{n} - 2n\right)^{n}$

For the following two questions determine the **non-negative** values of x such that the following functions are defined.

8) $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n!}$

9) $g(x) = \displaystyle\sum_{n=1}^{\infty} n^n x^n$

Problems to Turn In

1) Assume b_n is a sequence of positive numbers such that $\displaystyle\lim_{n\to\infty} b_n = \frac{1}{2}$. Explain why the following sequence converges

$$\sum_{n=1}^{\infty} \frac{b_n^n}{n}.$$

2) Determine the **non-negative** values of x such that the following function is defined

$$h(x) = \sum_{n=1}^{\infty} (\ln n) x^n.$$