

1) a. 6      b. 4      c.  $\sqrt{4^2+6^2} = \sqrt{52}$       d.  $\sqrt{4^2+(-2)^2+6^2} = \sqrt{56}$

2) Let  $A=(0,-5,5)$  so  $d(A,B) = \sqrt{(0-1)^2+(-5-(-2))^2+(5-4)^2} = \sqrt{11}$   
 $B=(1,-2,4)$        $d(A,C) = \sqrt{(0-3)^2+(-5-4)^2+(5-2)^2} = 3\sqrt{11}$   
 $C=(3,4,2)$        $d(B,C) = \sqrt{(1-3)^2+(-2-4)^2+(4-2)^2} = 2\sqrt{11}$

Since  $d(A,B)+d(B,C) = d(A,C)$ ,  
the points lie on the same line

3)  $(x-1)^2+(y-2)^2+(z+3)^2 = 16$

4) The distance between the points

$$\sqrt{(4-3)^2+(3-8)^2+(-1-(-1))^2} = \sqrt{1^2+5^2+0^2} = \sqrt{26}$$

will be the radius, so the sphere's equation is

$$(x-3)^2+(y-8)^2+(z+1)^2 = 26$$

5)  $x^2+y^2+z^2-2z-4y+8z = 0$       or  $x^2+y^2+z^2-2x-4y+8z = 0$   
 $\Leftrightarrow x^2+y^2-4y+z^2+6z = 0$        $\Leftrightarrow x^2-2x+1+y^2-4y+4+z^2+4z+16=21$   
 $\Leftrightarrow x^2+y^2-4y+4+z^2+6z+9 = 13$        $\Leftrightarrow (x-1)^2+(y-2)^2+(z+4)^2 = 21$   
 $\Leftrightarrow x^2+(y-2)^2+(z+3)^2 = 13$

6) The point halfway

$$\left(\frac{2+4}{2}, \frac{-1+5}{2}, \frac{1-4}{2}\right) = \left(3, 2, -\frac{3}{2}\right)$$

will be the center

Equation:  $(x-3)^2+(y-2)^2+(z+\frac{3}{2})^2 = \frac{65}{4}$

The radius will be

$$\frac{1}{2} \sqrt{(2-4)^2+(-1-5)^2+(1-(-4))^2}$$

$$= \frac{1}{2} \sqrt{4+36+25} = \frac{1}{2} \sqrt{65}$$

(this is half the diameter)

7) The closest coordinate plane is the xz-plane ( $y=0$ ),

so the sphere will have radius 1.

Equation:  $(x-5)^2+(y-1)^2+(z-9)^2 = 1$

8) a)  $x^2 + y^2 + z^3 = 4$  or, equivalently,  $x^2 + y^2 = 4 - z^3$

Let  $z$  be fixed. For  $z > \sqrt[3]{4}$ , no choice of  $x$  and  $y$  will satisfy the equation. At  $z = \sqrt[3]{4}$ ,  $x = y = 0$ .

For  $z < \sqrt[3]{4}$ , the possible  $x$  and  $y$  coordinates form a circle of radius  $\sqrt{4 - z^3}$ . As  $z$  decreases, this radius increases.

Piecing the circles together, we have a circular surface widening as its height decreases, coming to a peak at  $z = 4$ .

b)  $z = \sin(x)$

$z$  will range between 1 and -1 like a wave, according to the value of  $x$ .  $y$  can take any value, so our surface will be an endless wave rising and falling across the  $x$ -axis.

c)  $(y-1)^2 + (z+2)^2 = 4$

In the  $yz$ -plane, we have a circle of radius 2. Since  $x$  can take any value, this will form a cylinder of radius 2 around the  $x$ -axis.

