Math 8
Homework Set #1

Sequences

Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

1) $1,\ -\frac{1}{3},\ \frac{1}{5},\ -\frac{1}{7},\ \frac{1}{9},\dots$

2) $4,\ 10,\ 28,\ 82,\ 244,\dots$

Determine whether each of the following sequence converges, diverges, or diverges to infinity and explain your reasoning. If it converges, find the limit.

3) $\left\{1-\left(\frac{1}{3}\right)^n\right\}_{n=1}^{\infty}$

4) $0,1,0,0,1,0,0,0,1,\dots$

5) $\left\{\frac{n^4+4}{n^2+2}\right\}_{n=1}^{\infty}$

6) $\left\{e^{1/n}\right\}_{n=1}^{\infty}$

7) $\left\{\frac{\cos^2 n}{n}\right\}_{n=1}^{\infty}$

8) $\left\{n-\sqrt{n}\sqrt{n+1}\right\}_{n=1}^{\infty}$

9) $\left\{\frac{n^n}{n!}\right\}_{n=1}^{\infty}$

10) $\left\{\frac{5n^2-3n+1}{n^3+1}\right\}_{n=1}^{\infty}$

In class we discussed the advantages of representing functions as "infinite polynomials", which we call Taylor series. For example, we saw that

$$\sin\ x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\frac{x^9}{9!}+\cdots.$$

In fact any "infinite polynomial" will always look like

$$c_0+c_1x+x_2x^2+c_3x^3+\dots$$

where the c_n are just coefficients. As the terms in this infinite sum yield the sequence

$$c_0,\ c_1x,\ c_2x^2,\ c_3x^3,\dots$$

we will be very interested in understanding sequences of this form. The next few problems deal explicitly with such sequences.

10) For the following sequence determine the values of x, if any, that make the sequence convergent. What does it converge to? Explain your reasoning.

$$x,\ \frac{x^2}{2},\ \frac{x^3}{3},\ \frac{x^4}{4},\ \ldots$$

11) Assume the following sequence converges to some number L, find L.

$$\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots.$$

Problems to Turn In

1) Find the limit of the following sequence.

$$a_n = n \sin\left(\frac{1}{n}\right)$$

2) For the following sequence determine the values of x, if any, that make the sequence convergent. What does it converge to? Explain your reasoning.

$$x,\ \frac{x^2}{2!},\ \frac{x^3}{3!},\ \frac{x^4}{4!}, \ldots$$