Math 8

Final Exam Practice

Multivariable Calculus

1) Find the values of x such that the vectors $< 3, 2, x >$ and $< 2x, 4, x >$ are orthogonal.

2) Find two unit vectors that are orthogonal to both $< 0, 1, 2 >$ and $< 1, -2, 3 >$.

3) Let $A = (1, 0, 0)$, $B = (2, 0, -1)$, and $C = (1, 4, 3)$ be points in 3-space.
   a. Find a vector perpendicular to the plane containing the points A, B, and C.
   b. Find the area of triangle ABC.

4) Find an equation of the line...
   a) that contains the points $(4, -1, 2)$ and $(1, 1, 5)$.
   b) that contains the point $(1, 0, -1)$ and is perpendicular to the plane $2x - y + 5z = 12$.

5) Find the equation of a plane...
   a) that contains the points $(2, 1, 0)$, $(4, 0, 2)$, and $(6, 3, 1)$.
   b) that contains the point $(1, 2, -2)$ and contains the line $x = 2t$, $y = 3 - t$, $z = 1 + 3t$.

6) Find the length of the curve $r(t) =< 2t^{3/2}, \cos 2t, \sin 2t >$, where $0 \leq t \leq 1$.

7) Find the equation of the line tangent to the curve $r(t) =< \frac{t^3}{3}, \frac{t^2}{2}, t >$, when $t = 1$.

8) Find the equation of the tangent plane to the surface $z = e^x \cos y$ at the point $(0, 0, 1)$.

9) Find the local maximum and minimum values and saddle point of the function

$$f(x, y) = (x^2 + y)e^{y/2}.$$

10) Find the absolute maximum and minimum values of $f(x, y) = 4xy^2 - x^2y^2 - xy^3$ on the closed triangular region in the xy-plane with vertices $(0, 0)$, $(0, 6)$, and $(6, 0)$.

11) A package in the shape of a rectangular box can be mailed by the US Postal Service if the sum of its height and girth (the perimeter of its base) is at most 108 in. Find the dimensions of the package with largest volume that can be mailed?

12) Let $x + 2y - 7z = 0$ be the equation of a plane and let A be the point $(5, 12, -19)$. Find the distance from the point A to the plane...

a) using scalar projection. (It might be helpful to review HW #4.)

b) using partial derivatives and critical points.

c) using Lagrange Multipliers.