Math 8 Assignment 9

Due: Start of class on Wednesday June 2.

Find the equation(s) of the plane(s) tangent to the surface given by

$$xy^2z^2 = 1$$

which are farthest from the origin, $(0, 0, 0)$.

This problem reviews **a lot** of material we've covered this term. You'll be making a lot of calculations, writing down functions, solving for variables, etc. Be careful to explicitly state what each of your functions and variables represent and don't reuse variable names if you can help it.

Some hints/guidelines:

- Find the tangent plane at a general point, say (x_0, y_0, z_0) using the method in §15.6.

- Use the distance formula from Chapter 13 to find the distance from the origin to this general plane.

- You should be able to reduce this formula to one with only two variables, using the fact that (x_0, y_0, z_0) is on the surface.

- The maximum of the distance function happens at the same point as the maximum of the distance squared.

- You should be able to get two equations involving your two variables, make a substitution, and then solve for each of the three variables easily.

- You shouldn't need a computer or calculator to take any of the derivatives or solve any of the equations, but it isn't a bad idea to check.