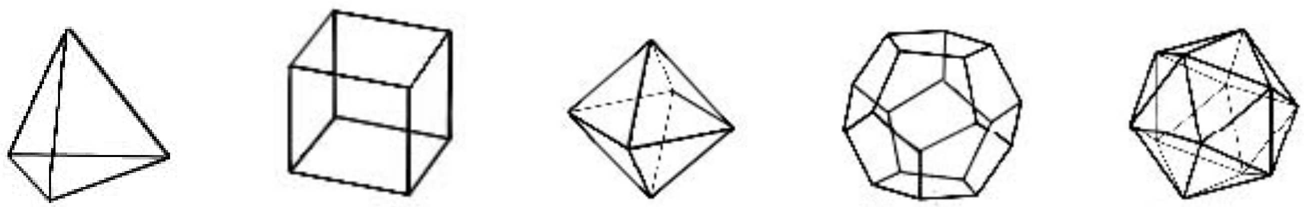
Math 8 Assignment 6

Due: Start of class on Friday May 7.

You may recall that regular polygons are those shapes whose sides all have equal length and whose interior angles are all congruent. The three-dimensional analogue of a regular polygon is something called a Platonic solid, discovered by the ancient Greek mathematician Plato. The faces of the Platonic solids are congruent regular polygons and the same number of faces meet at each vertex. So all edges are congruent as well all angles between adjacent edges. There are only five Platonic solids:



(a) Tetrahedron (b) Cube (c) Octahedron (d) Dodecahedron (e) Icosahedron

Figure 1: The Five Platonic Solids

The simplest of these is the regular tetrahedron. A regular tetrahedron has four faces, each of which is an equilateral triangle.

The base of a tetrahedron can be represented by two vectors **a** and **b** based at the origin in the xy plane (where the line segment connecting the heads of the two vectors forms the third side of the triangle). To represent the third dimension of the tetrahedron, we need only one more vector, **c**, based at the origin. (Connecting the head of **c** with the heads of **a** and **b** provides the other two edges.) Use your knowledge of vectors to calculate **a**, **b** and **c** for a regular tetrahedron built from equilateral triangles with side length 1.

The volume of a tetrahedron is given by the formula

$$V = \frac{1}{3}Ah$$

where A represents the area of the base and h represents the height. Calculate the volume of the regular tetrahedron you described above.

Recall that you can define a parallelepiped by three vectors, **a**, **b**, and **c**, and its volume is given by a formula involving the dot and cross products of **a**, **b**, and **c**. Calculate the volume of the parallelepiped defined by your three vectors **a**, **b**, and **c**.

How many tetrahedrons can you fit inside a parallelepiped?