Amy Burzynski
4/15/10
Math 8 Written Assignment 3 CREDIT Super Great!

Approximate the constant π to six digits using Approximate Integration methods and error estimates.

Solution:

First I found an integral to evaluate that would equal π.

$\tan(\frac{\pi}{4}) = 1$

$\arctan(1) = \frac{\pi}{4}$

$4\arctan(1) = \pi$

$(4\arctan(x))' = 4\left(\frac{1}{1+x^2}\right)dx = \frac{4}{1+x^2}$

$a = 0$ ← maybe add that you chose this because $4\arctan(0) = 0$

$b = 1$

$$\int_0^1 \frac{4}{1+x^2}\,dx \longrightarrow \text{will equal } \pi$$

$$= [4\arctan(x)]_0^1$$
$$= \pi - 0$$
$$= \pi$$

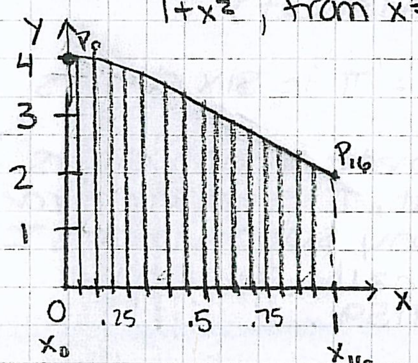
From the approximate integration methods presented in section 8.7 in the text, I chose to use Simpson's Rule.

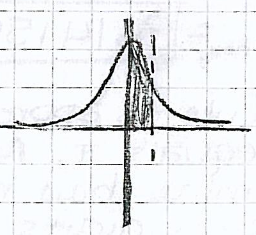
$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

where n is even and $\Delta x = \frac{(b-a)}{n}$

Simpson's rule uses parabolas to approximate a curve:

$f(x) = \frac{4}{1+x^2}$, from $x=0$ to $x=1$





To find the number of subdivisions, n, that are needed to ensure $\int_0^1 \frac{4}{1+x^2}dx$ is accurate to six digits (0.00001) we can use the Error Bound:

$|f^{(4)}(x)| \le B$ for $a \le x \le b$ $a = 0$ $b = 1$ E_s is error in Simpson's Rule

$0 \le x \le 1$

$$|E_s| \le \frac{B(b-a)^5}{180n^4}$$

To find B, we need to calculate $f^{(4)}(x)$:

$$f(x) = \frac{4}{1+x^2} \qquad f'(x) = \frac{8x}{(1+x^2)^2} = \frac{8x}{x^4+2x^2+1}$$

$$f''(x) = \frac{(x^4+2x^2+1)(8x)' - (8x)(x^4+2x^2+1)'}{(x^4+2x^2+1)^2}$$ Quotient Rule

$$= \frac{(x^4+2x^2+1)(8) - (8x)(4x^3+4x+0)}{(x^4+2x^2+1)^2}$$

$$= \frac{8x^4+16x^2+8-32x^4+32x^2}{(x^4+2x^2+1)^2}$$

$$f''(x) = \frac{-24x^4+48x^2+8}{(x^4+2x^2+1)^2}$$

$$f'''(x) = \frac{96x^3-96x}{x^8+4x^6+6x^4+4x^2+1}$$

$$f^{(4)}(x) = \frac{480x^4-960x^2+96}{x^{10}+5x^8+10x^6+10x^4+5x^2+1}$$

I found the third and fourth derivatives online to ensure accuracy, but to do them out by hand I would use the Quotient Rule as I did for $f''(x)$.

where is $f^{(4)}(x)$ at its maximum? at $x=0$

$B = f^{(4)}(0) = 96$

$$|E_S| \leq \frac{B(b-a)^5}{180n^4}$$

$$0.00001 \leq \frac{96(1-0)^5}{180n^4}$$

$$0.0018 = \frac{96}{n^4}$$

$n \approx 15.1967$

For Simpson's Rule, n must be even, so I will use 16 subdivisions. I calculated $S_{n=16}$ in Excel (see attached spreadsheet)

I found that $\int_0^1 \frac{4}{1+x^2}\,dx \approx S_{n=16} = \boxed{3.14159} = \pi$ to six digits

In conclusion, I used Simpson's Rule to approximate π to six digits. First, I found an integral that equals π. Next, I used the error bound for Simpson's Rule to determine how many subdivisions I would need to use to find π to six digits. Finally, I used Simpson's Rule in Excel to calculate $\pi = 3.14159$.

Why do we care?

π is an irrational number, but we can approximate it through integration. (with accuracy determined by the number of subdivisions used).

LOVE IT!

Simpson's Rule

x	f(x)	multiplied by	for Sn
0	4.00000	1	4.00000
0.0625	3.98444	4	15.93774
0.125	3.93846	2	7.87692
0.1875	3.86415	4	15.45660
0.25	3.76471	2	7.52941
0.3125	3.64413	4	14.57651
0.375	3.50685	2	7.01370
0.4375	3.35738	4	13.42951
0.5	3.20000	2	6.40000
0.5625	3.03858	4	12.15430
0.625	2.87640	2	5.75281
0.6875	2.71618	4	10.86472
0.75	2.56000	2	5.12000
0.8125	2.40941	4	9.63765
0.875	2.26549	2	4.53097
0.9375	2.12890	4	8.51559
1	2.00000	1	2.00000
		Sum	150.79645
		*deltax/3	**3.14159**
deltax/3=	0.0208		