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Math 8 — Assignment 1

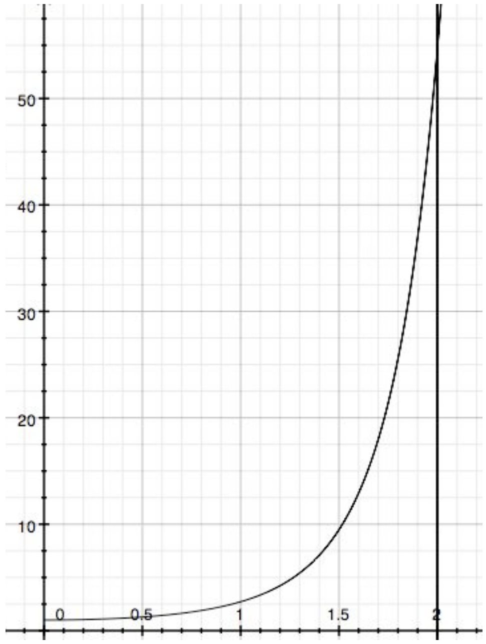
1. The integral $\int_{1}^{e^4} \pi(4 - \ln y)dy$ could be the result of a volume of revolution. This integral looks much like $\int \pi(R^2 - r^2)h$, which is the general form for the volume of a solid of revolution using the washer method. Based on the similarities, we can figure out what solid of revolution would result in the original integral.

- Firstly, *dy* is equivalent to *h*. This tells us that the washers are being cut horizontally and that the area must be rotated about the y-axis.
- 4 is analogous to R^2, the outer radius. One boundary, therefore is:
  - $R^2 = 4 \quad \therefore \quad R = 2 \quad so\ldots \quad x = 2$
- lny is analogous to r^2, the inner radius. A second boundary is:
  - $R^2 = \ln y \quad so\ldots \quad x = \sqrt{\ln y}$
- Because the first limit of integration is y=1 and $\sqrt{\ln(1)} = 0$, a third boundary is x=0.
- The last boundary is y=1. This is the lower limit of integration.

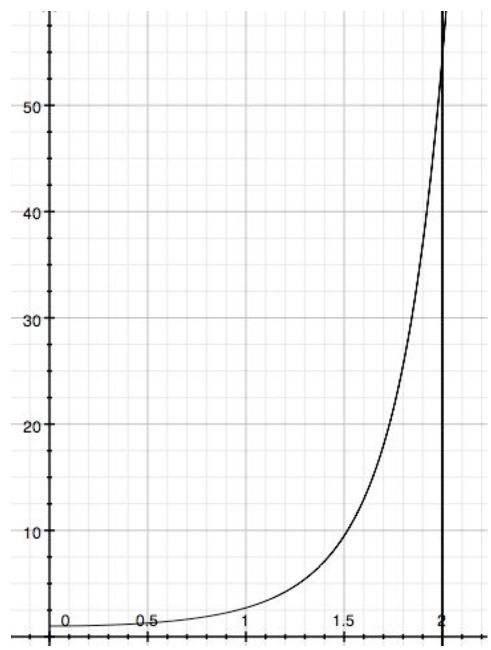
The solid of revolution is produced rotating the area bounded by $x = 2$, $x = \sqrt{\ln y}$, x=0, and y=1 around the y-axis.

2. A second method of evaluating this same volume would be the cylinder method. To do this we must find a circumference, a height, and a thickness so that we can use $\int 2\pi rh(thickness)$.

- The upper limit of the height is the function $x = \sqrt{\ln y} \quad \therefore \quad y = e^{x^2}$. Its lower boundary is y=1. Therefore, the entire height function is $y = e^{x^2} - 1$
- The thickness is *dx*.
- The radius is x.
- The limits of integration are x=0 and x=2, which are boundaries from the original solid.



Slicing vertically gives many cylindrical shells whose volume (effectively their area), can be summed using the above integral



The new definite integral is: $2\pi\int_0^2 (x)(e^{x^2}-1)dx = 2\pi\int_0^2 \left(xe^{x^2}-x\right) dx$

3. To solve the integral, $2\pi\int_0^2\left(xe^{x^2}-x\right)dx$, we must first split the integral and then use u-substitution:

$$2\pi\int_0^2\left(xe^{x^2}-x\right)dx=2\pi\int_0^2\left(xe^{x^2}\right)dx-2\pi\int_0^2(x)\,dx$$

Now that the integral has been split, we will integrate the first half of it using u-substitution:

$$2\pi\int_0^2(x)e^{x^2}dx \quad let\ u=x^2 \quad du=2x\ dx$$

$$\pi\int_{x=0}^{x=2}e^u du$$

This gives an easily integrated function:

$$\pi\int_{x=0}^{x=2}e^u du=\pi\left[e^u\right]_{x=0}^{x=2}$$

We then replace u with the function it represents:

$$\pi\left[e^u\right]_{x=0}^{x=2}=\pi\left[e^{x^2}\right]_0^2$$

Now we must reintroduce the second half of the original integrand, $-2\pi\int_0^2(x)\,dx$.

This integrates using the power rule to $-\pi\left[x^2\right]_0^2$. Combining the two integrated functions gives:

$$\pi\left[e^{x^2}-x^2\right]_0^2$$

$$=\pi\left(e^4-4\right)-\pi\left(e^0-0\right)=\pi\left(e^4-4-\pi\right)$$

$$\boxed{=155.817}$$