Math 8: Mock Quiz

Question

Does the series

$$\sum_{n=1}^{\infty} \frac{3}{n+2^n}$$

converge or diverge? Justify your answer using one or more of the convergence tests that were presented in lectures.

(Possible) Answer

The nth term of the given series is

$$a_n = \frac{3}{n+2^n}.$$

We compare with the series

$$\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^n,$$

and let $b_n = \frac{3}{2^n}$ denote its nth term. Since this is a geometric series with ratio $\frac{1}{2} < 1$, it converges. As $0 \le a_n \le b_n$, we can apply the Direct Comparison Test which shows that the given series converges.

Alternative

Since

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{3}{n+2^n}\,\frac{2^n}{3} = \lim_{n\to\infty} \frac{1}{\frac{n}{2^n}+1} = 1,$$

the Limit Comparison Test shows that the given series converges. [In order to see

$$\lim_{n\to\infty} \frac{n}{2^n} = 0,$$

use l'Hospitals rule together with

$$(2^x)' = \left(e^{\ln 2^x}\right)' = \left(e^{x\,\ln 2}\right)' = \ln 2\,\left(e^{x\,\ln 2}\right) = \ln 2\, 2^x]$$

Question

Does the series

$$\sum_{n=1}^{\infty} (-1)^{n+1}\,\frac{3n-12}{n^2}$$

converge or diverge? Justify your answer using one or more of the convergence tests that were presented in lectures.

(Possible) Answer

The given series can be viewed as the sum of the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n+1}\,\frac{3}{n} \qquad \text{and} \qquad \sum_{n=1}^{\infty} (-1)^{n}\,\frac{12}{n^2}.$$

Both series pass the Alternating Series Test since the sequences given by

$$a_n = \left|(-1)^{n+1}\,\frac{3}{n}\right| = \frac{3}{n} \qquad \text{and} \qquad b_n = \left|(-1)^{n}\,\frac{12}{n^2}\right| = \frac{12}{n^2}$$

are positive sequences that decrease to 0. Therefore, these alternating series converge. Hence, their sum – which is the given series – is also convergent.