MATH 8 FALL 2010 MIDTERM 1 REVIEW SHEET

This sheet contains a quick summary of definitions, tests, and other facts which might be useful for the second midterm. There are no examples on this sheet, and is only intended to be a convenient reference where you can see all the basic ideas of the past four weeks in a single place. The following lists may not be comprehensive – it is possible that I forgot to include certain definitions or theorems!

1. Tests for series

- Know how to evaluate the limit of a sequence.
- Know that a series is defined as the limit of the sequence of its partial sums.
- The nth term test: if $\lim_{n\to\infty} a_n \neq 0$ (this includes cases where the limit does not exist), then $\sum a_n$ diverges.
- Geometric series: A geometric series is a series of the form $a+ar+ar^2+ar^3+\ldots, a \neq 0$, this series converges if $|r| < 1$, diverges if $|r| \geq 1$. If the series converges, its value is given by $\dfrac{a}{1-r}$.
- Integral test: if $\sum a_n$ is a sequence with $a_n = f(n)$, where $f(x)$ is a function defined on $[1,\infty)$, and is positive, continuous and decreasing, then $\sum a_n$ converges exactly when the integral $\int_1^\infty f(x)\,dx$ converges.
- Error term in integral test: Suppose $\sum a_n$ passes the integral test with $f(x)$ as above, and let s be the value of this series. Let $s_n = a_1 + \ldots + a_n$ be the nth partial sum. Then the error in approximating s with s_n satisfies inequalities
$$\int_{n+1}^\infty f(x)\,dx \leq |s - s_n| \leq \int_n^\infty f(x)\,dx.$$
- Direct comparison test: if $0 \leq a_n \leq b_n$ for all n, then $\sum a_n$ converges if $\sum b_n$ converges. If $\sum a_n$ diverges, then $\sum b_n$ diverges. You cannot conclude anything if $\sum b_n$ diverges or if $\sum a_n$ converges.
- Limit comparison test: if $a_n, b_n > 0$, and if $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} = L$, where L is a nonzero number, then $\sum a_n, \sum b_n$ either converge at the same time or diverge at the same time.
- Alternating series test: consider the alternating series $\sum(-1)^n a_n$, where $a_n > 0$. Then this series converges if $\lim_{n\to\infty} a_n = 0$ and if a_n is a decreasing sequence.
- Error term in the alternating series test: Suppose $\sum(-1)^n a_n$ converges by the alternating series test and has value s. Let $s_n = -a_1 + a_2 - a_3 + \ldots + (-1)^n a_n$ be the nth partial sum (if the alternating series starts with a positive term, flip the signs of every term above). Then
$$|s_n - s| \leq a_{n+1}.$$
- Ratio test: suppose $\sum a_n$ is a series with all terms nonzero. If
$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L,$$
then $\sum a_n$ converges if $L < 1$, diverges if $L > 1$ or if the limit diverges to infinity, and is inconclusive if $L = 1$ or if the limit diverges, but not to infinity.

- Power series: Know how to use the ratio test and other tests to find the interval of convergence of a power series $\sum c_n(x-a)^n$.
- Know that a power series has an interval of convergence centered around a, of radius R, and that in general one needs to test for convergence/divergence at the endpoints, $x = a \pm R$, using something besides the ratio test.
- Know how to manipulate power series by multiplying a series by a power of x, replacing x with cx for some nonzero c, or replacing x by a power of x.
- Recognize how certain power series are geometric series and their value can be computed (as a function of x), and vice versa, know how to recognize certain functions as values of power series.
- Know how to integrate and differentiate power series term by term. Also know that this operation does not change the radius of convergence, but it might change the interval of convergence by affecting what happens at the endpoints of the interval.

2. Tips for preparing

The following suggestions may or may not help you prepare for the test. (This is identical advice to that given on the last review sheet.)

- Take a look at all your old homework assignments, and learn how to solve them all by yourself. Many people either received assistance or took many attempts to correctly answer questions – at this point, you should be able to solve every old homework problem, correctly, on your own.
- For additional practice problems, try solving random questions from the text.
- Make sure you are precise with arithmetic and algebra. Being able to do both these accurately, by hand, is an important skill to cultivate.
- Make sure you are well-rested and not hungry during the exam. You may also want to bring a light coat, in case the room is too cold. Also make sure you bring enough pencils or pens to the test site.
- This isn't a tip, but PLEASE turn off all cell phones and other portable electronic devices during the exam. Even phones on vibrate settings are a nuisance to nearby test takers.
- Try to keep the time limit in mind as you take the exam. Do not spend too much time obsessing about a small number of problems if it will be detrimental to your performance on the remaining problems.
- Show all your work, and try to be organized and neat when you write your solutions. This will make it easier for the graders to follow your line of thought, and may help you earn (more) partial credit if you cannot completely solve a question correctly.