1. (16) Find the radius of convergence of each of the following power series. (You do not have to find the interval of convergence, only the radius.)

(a)

$$\sum_{n=0}^{\infty} \frac{n4^n(x-2)^n}{n!}$$

(continued on next page)

(b)

$$\sum_{n=1}^{\infty} \frac{2^{2n+1}(x-2)^n}{\sqrt{n}}$$

2. (12) For each of the following series, determine whether the Alternating Series Test may be used to show convergence. If so, justify your answer. If not, indicate which of the conditions of the test fails to hold.

(a)

$$\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{2} + \frac{1}{n}\right)$$

(b)

$$1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{5} + \frac{1}{4} - \frac{1}{7} + \frac{1}{6} - \dots$$

3. (15) (12 + 5 points)

(a) Write down the degree three Taylor polynomial (i.e., the first four terms of the Taylor series) of

$$f(x) = \frac{1}{\sqrt{x}}$$

centered about $a = 1$.

(b) Let $\sum_{n=0}^{\infty} c_n(x-1)^n$ be the Taylor series for $f(x) = \frac{1}{\sqrt{x}}$ centered about $a = 1$. This series converges to $f(x)$ on its interval of convergence. Determine whether the series converges at $x = 3$. Justify your answer. (Note: It's possible to do this *without* actually finding the Taylor series and without using the ratio test. Hint: What is the domain of the function f?)

4. (12)

Find the Maclaurin series with constant term zero for the integral

$$\int x \sin(x^3)\, dx$$

To receive full credit, you should give the entire series. However, you will receive partial credit by giving the first three terms correctly.

5. (13) (7 + 6 points) Suppose that the partial sums of the series $\sum_{n=1}^{\infty} a_n$ are given by $s_n = \frac{n}{n+1}$.

(a) Does the series $\sum_{n=1}^{\infty} a_n$ converge? If so, find its sum. If not, explain why.

(b) Find a_7.

6. (12) Consider the series

$$-\frac{1}{2}+\frac{1}{8}-\frac{1}{48}+\cdots=\sum_{n=1}^{\infty}\frac{(-1)^n}{2^n n!}$$

(a) Find the sum of the series.

(b) Suppose we approximate the value of this series with one of its partial sums. What is the least number of terms we can use and still insure that the error is less than $\frac{1}{1000}$?

7. (20) (4 points each) Multiple choice. Circle the correct statement. No partial credit will be given. There is no need to justify your answer.

(a) The sequence $\left\{ \left(\frac{e}{40} \right)^n \right\}$

A. converges to 0

B. converges to $\dfrac{1}{1 - (\frac{e}{40})}$

C. has no limit

D. none of the above

(b) Which of the following statements is correct?

A. Every bounded sequence converges.

B. Every decreasing sequence converges.

C. Every convergent sequence is bounded.

D. None of the above.

(c) Which of the following sequences converge?

A. $\{(-1)^n \frac{n}{n+1}\}$
B. $\{3^n/n!\}$
C. $\{n^n/n!\}$
D. {All of the above}

(d) Consider the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^{3/2}}.$$

A. The ratio test can be used to show that the series converges absolutely.
B. The alternating series test can be used to show that the series converges.
C. The nth term test (also called the divergence test) can be used to show that the series diverges.
D. The series is a convergent geometric series.

(e) Which of the following statements is true for *every* divergent series $\sum_{n=1}^{\infty} a_n$?

A. $\lim_{n\to\infty} a_n \neq 0$

B. The sequence of partial sums approaches ∞.

C. The series $\sum_{n=1}^{\infty} |a_n|$ is divergent

D. None of the above.

NAME : ______________________________

SECTION : (circle one) Yang Gordon Arkowitz

Math 8

15 October 2009

Hour Exam I

INSTRUCTIONS: This is a closed book exam and no notes are allowed. You are not to provide or receive help from any outside source during the exam except that you may ask either of the instructors for clarification. You have two hours and you should attempt all problems.

- Wait for signal to begin.
- ***Print*** your name in the space provided and circle your instructor's name.
- Sign the FERPA release below *only if* you wish your exam returned in lecture.
- Calculators or other computing devices are not allowed.
- Except for problem #7, you must show your work and justify your assertions to receive full credit.

FERPA RELEASE: Because of privacy concerns, we are not allowed to return your graded exams in lecture without your permission. If you wish us to return your exam in lecture, please sign on the line indicated below. Otherwise, you will have to pick your exam up in your instructor's office after the exams have been returned in lecture.

SIGN HERE: ______________________________________.

Problem	Points	Score
1	16	
2	12	
3	15	
4	12	
5	13	
6	12	
7	20	
Total	100	