How Many Angels?
Philosophy, Mathematics and the Infinite

Problems to Solve Using Infinitesimals

(1.) A spherical cloud has a radius r that is changing at a rate $\dot{r}$. If the total mass in the cloud remains constant, at what rate is the mass density changing?

(2.) A cylindrical region has a variable shape but a constant volume. If the radius r of the base of the cylinder is changing at a rate $\dot{r}$, at what rate is the height of the cylinder changing?

(3.) A rectangular box has a variable shape but constant volume. If the lengths a and b of two of its dimensions are changing at rates $\dot{a}$ and $\dot{b}$, at what rate is its third dimension changing?

(4.) A (right circular) cone has a variable shape but constant volume. If the height h of the cone is changing at a rate $\dot{h}$, at what rate is the radius of its circular base changing? (The volume of a right circular cone is one third the product of its height with the area of its base.)

(5.) A right triangle has a hypoteneuse of constant length. If the length a of one of its legs is changing at a rate $\dot{a}$, at what rate is the length of the other leg changing?

(6.) Two points P_1 and P_2 are moving along lines that form a 45° angle with each other. If P_1 is at a distance x_1 from the intersection of the two lines, P_2 is at a distance x_2 from the intersection of the two lines, and x_1 is changing at a rate $\dot{x}_1$, at what rate must x_2 change in order for the distance between the two points to remain constant?

(7.) You are equidistant between two sources of radiation, one twice as strong as the other. The intensity of radiation from each source is inversely proportional to the square of your distance from that source. In what direction should you move in order for your total radiation exposure to decrease as quickly as possible?