Name: KEY

# Math 6 Midterm Exam

July 27, 2005

This is a closed book, closed notes exam. You are on your honor not to use outside sources on this exam. You are also on your honor not to talk to another student about this exam until both you and the other student have handed in the exam. Show all your work, and clearly indicate your answers to each problem.

| Problem | Points Possible | Points Earned |
|---|---|---|
| 1 | 8 | |
| 2 | 6 | |
| 3 | 10 | |
| 4 | 6 | |
| 5 | 12 | |
| 6 | 8 | |
| Bonus | 3 | |
| Total | 50 | |

1. (8 points) Let $p$ denote the statement "Dartmouth College is in Utah," and let $q$ denote the statement "Some roses are red."

   (a) Write each of the following statements in proper English, and determine its truth value.

      i. $p \wedge q$

      Dartmouth College is in Utah, and some roses are red.

      False (Dartmouth College is not in Utah)

      ii. $\sim q \rightarrow p$

      If no roses are red, then Dartmouth College is in Utah.

      true (it is not true that no roses are red)

   (b) Put each of the following statements into symbolic form, and determine its truth value.

      i. Dartmouth College is in Utah, or some roses are red.

      $p \vee q$

      true (it is true that some roses are red)

      ii. Dartmouth College is in Utah, or some roses are red, but not both.

      $p \oplus q \equiv (p \vee q) \wedge \sim (p \wedge q)$

      true (Dartmouth College is not in Utah, but some roses are red)

2. (6 points) Show that the following argument is valid.

If the price of oil increases, the OPEC countries are in agreement. If there is no U.N. debate, the price of oil increases. The OPEC countries are in disagreement. Therefore, there is a U.N. debate.

let $p$ denote the statement "the price of oil increases," $a$ the statement "the OPEC countries are in agreement," and $d$ the statement "there is a U.N. debate."
then we have the following direct argument:

| | |
|---|---|
| 1. $\sim a$ | hypothesis |
| 2. $p \to a$ | hypothesis |
| 3. $\sim p$ | modus tollens (1,2) |
| 4. $\sim d \to p$ | hypothesis |
| 5. $\sim(\sim d)$ | modus tollens (3,4) |
| 6. $d$ | double negation (5) |

alternatively, we have the following indirect argument:

$H_1: p \to a$
$H_2: \sim d \to p$
$H_3: \sim a$
$C: d$

| | |
|---|---|
| 1. $\sim d$ | $\sim C$ |
| 2. $\sim d \to p$ | $H_2$ |
| 3. $p$ | modus ponens (1,2) |
| 4. $p \to a$ | $H_1$ |
| 5. $a$ | $\sim H_3$, modus ponens (3,4) |

One can also use a truth table:

[illegible]

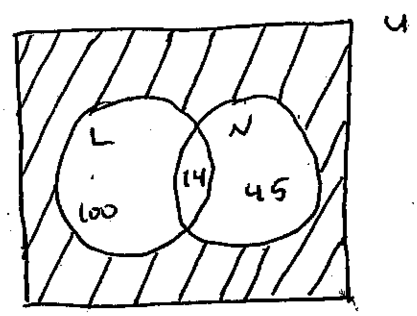
| $p$ | $a$ | $d$ | $p \to a$ | $\sim d$ | $\sim d \to p$ | $\sim a$ | $(p \to a) \wedge (\sim d \to p) \wedge (\sim a)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | F | T | F | F |
| T | T | F | T | T | T | F | F |
| T | F | T | F | F | T | T | F |
| T | F | F | F | T | T | T | F |
| F | T | T | T | F | T | F | F |
| F | T | F | T | T | F | F | F |
| F | F | T | T | F | T | T | T |
| F | F | F | T | T | F | T | F |

since $(p \to a) \wedge (\sim d \to p) \wedge (\sim a)$ only has truth value true when $d$ has truth value true, $(p \to a) \wedge (\sim d \to p) \wedge (\sim a)$ <u>logically implies</u> $d$, so the argument is valid.

3. (10 points) A total of 750 Nobel Prizes had been awarded by 2002. Fourteen of the 100 prizes in literature were awarded to Scandinavians. Scandinavians received a total of 45 awards.

(a) Draw a Venn diagram displaying the given data. Shade the region corresponding to the set of Nobel Prizes outside of literature that have been awarded to non-Scandinavians.

L = Nobel prizes awarded in literature
N = Nobel prizes awarded to Scandinavians




$n(U) = 750$
$n(L) = 100$
$n(L \cap N) = 14$
$n(N) = 45$

(b) How many Nobel Prizes outside of literature have been awarded to non-Scandinavians?

$n(L \cup N) = n(L) + n(N) - n(L \cap N)$ by the inclusion-exclusion principle

$= 100 + 45 - 14 = 145 - 14 = 131$

$n((L \cup N)') = 750 - n(L \cup N)$

$= 750 - 131 = \underline{619}$

4. (6 points) The Buildings Society has 17 members.

(a) In how many ways can the members elect a president, vice-president, treasurer, and secretary if no person can have more than one position?

| | |
|---|---|
| T1: elect president | 17 |
| T2: elect vice-president | 16 |
| T3: elect treasurer | 15 |
| T4: elect secretary | 14 |

# ways to elect president, vice-president, treasurer, and secretary if no person can have more than one position

$= \underline{\underline{17 \cdot 16 \cdot 15 \cdot 14}} = \underline{\underline{P(17,4)}} = \underline{\underline{57120}}$

(b) In how many ways can the members select three people to be in charge of recruiting if the president, vice-president, treasurer, and secretary are not eligible?

order does <u>not</u> matter $\underline{\underline{\binom{13}{3}}} = \underline{\underline{C(13,3)}} = \underline{\underline{286}}$

(c) In how many ways can the Buildings Society elect a president, vice-president, treasurer, and secretary and select the three people to be in charge of recruiting?

| | |
|---|---|
| T1: elect president | 17 |
| T2: elect vice-president | 16 |
| T3: elect treasurer | 15 |
| T4: elect secretary | 14 |
| T5: select recruiting committee | $\binom{13}{3}$ |

$\underline{\underline{17 \cdot 16 \cdot 15 \cdot 14 \cdot \binom{13}{3}}} = \underline{\underline{P(17,4) \cdot C(13,3)}} = \underline{\underline{16,336,320}}$

5. (12 points) Say whether each of the following statements is true (T) or false (F).

(a) T $p \to q \equiv \sim q \to \sim p$ contrapositive

(b) F $p \to q \equiv q \to p$ take $p$ true, $q$ false

(c) T Two sets $A$ and $B$ such that $A \cap B = \emptyset$ are disjoint.

(d) T If $A$ and $B$ are disjoint sets, then $n(A \cup B) = n(A) + n(B)$.

(e) F A permutation of $n$ objects taken $r$ at a time is an unordered arrangement of $r$ of the $n$ objects.

(f) T A combination of $n$ objects taken $r$ at a time is an unordered arrangement of $r$ of the $n$ objects.

6. (8 points) For each of the following, indicate if the sentence is a statement.

(a) He is a brave fellow.

not a statement — "he" is not specified

(b) The number $\sqrt{5}$ is rational.

Statement

(c) If snow falls on the Rockies, people are skiing in Aspen.

statement

(d) This sentence is false.

not a statement — neither true nor false

**Bonus** (3 points): Suppose you are the only person working at the front desk of Hilbert's Hotel. Currently, all the rooms are booked, and you are looking forward to a relaxing evening when Harry Potter and the six other members of his Quidditch team enter. Is there a way to give each of these seven people a room? If so, in which rooms do you put each of them, and in which rooms do the current guests stay?

there <u>is</u> a way to give each of the seven people a room
put the guest currently in room 1 in room 8,
the guest currently in room 2 in room 9,
the guest currently in room 3 in room 10, and in general,
the guest currently in room $n$ in room $n+7$.
then put Harry and his friends in rooms 1 through 7.