

Mathematics 5
Winter Term 2006
The World According to Mathematics

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Friday Discussion: Week #2

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Today we are going to discuss how we reach conclusions and make decisions.

Advertising agencies often rely on the consumer's lack of logic in developing their advertising statements. The idea is to make a statement that will result in most consumers viewing the product in a favorable light. In fact, the conclusions reached by consumers may not be valid. Here are two examples.

1. An automobile advertisement states: "85 percent of all our trucks sold in the United States during the past 10 years are still on the road." What are some of the conclusions that a consumer may reach from this advertisement?

1. A tuna advertisement states: "How come only one leading tuna has the government seal of approval?" What are some conclusions the consumer is encouraged to draw?

The next problems are taken from a book called "What is the Name of This Book?" by Raymond Smullyan. The problems concern the island of knights and knaves. On this island, every inhabitant is either a knight or a knave. Knights always tell the truth. Knaves never tell the truth; any sentence uttered by a knave is false. You are a tourist on the island and you stop to talk to some of the inhabitants.

1. You meet three inhabitants of the island, A, B, and C. You ask A, "Are you a knight or a knave?" A answers, but he mumbles and you can't hear what he says. You ask B, "What did A say?" B replies, "A said that he is a knave." C then says, "Don't believe B, he is lying!" What are B and C?

2. You travel a little farther and meet another three inhabitants, A, B, and C. This time you ask A, "How many knights are among you?" but again you can't make out his reply. So you ask B, "What did A say?" and B replies, "A said there is one knight among us." Again C says, "Don't believe B, he is lying!" Again, what are B and C?

3. A little farther on in your travels you meet two people, A and B. A tells you, "At least one of us is a knave." What are A and B?

4. Suppose A had said, "Either I am a knave or B is a knight." What are A and B?

5. You travel on and meet an inhabitant who says, "Either I am a knave or else two plus two equals five." What should you conclude?

The next problems raise more interesting issues in decision-making.

1. **The Problem of the Light Switch:** I have an ordinary light switch connected to a light. When the switch is closed, the light is on. When the switch is open, the light is off. At two minutes to noon, the light is on. At one minute to noon I flip the switch, turning the light off. At half a minute to noon I flip it again, turning the light on. At a quarter of a minute before noon I flip it again, turning the light off. I continue in this way, cutting the time between flippings of the switch in half each time. Now this will be an infinite series of flips. The switch flippings will occur closer and closer to noon, but will all be completed before noon. Will the switch be on or off at noon?
2. **The Super Bullet:** The Acme Arms Company has invented a Super Bullet: a Super Bullet penetrates anything it hits. But the Adamantine Armor company has invented a Super Strong Armor Plate: nothing that hits a Super Strong Armor Plate penetrates it. The army is planning to shoot a Super Bullet at a Super Strong Armor Plate. What will happen?
3. **The Surprise Test:** One day the professor came into the class and announced, "Next week I will give you a surprise test. It will be a surprise, because you won't be able to figure out on which day it will occur until the class meets on the day of the test. It could happen on Monday, Tuesday, Wednesday, Thursday, or Friday, but I won't tell you which day."

The class was clever. They reasoned as follows: "She can't give the test on Friday, because then it wouldn't be a surprise; we'd know after class on Thursday that the test hadn't yet occurred, and hence we'd figure out that it would have to be on Friday. So we know the test can't be on Friday. But then it can't be on Thursday either, because if it were, we would know after class on Wednesday that it would have to be on Thursday, since it wouldn't have happened yet, and we have already shown that it can't be on Friday." Reasoning in this manner, the students concluded that the test could not occur on Wednesday either, nor on Tuesday, nor on Monday. Having concluded that a surprise test was impossible, the students didn't study. They were very disappointed and very surprised on Wednesday when they got a test. Where did the students' reasoning go wrong?