

INTRODUCTION

Our "Coin" and the Language of Hypothesis Testing

Math 5 Crew

Department of Mathematics

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The Null Hypothesis

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- Our Example: "Heads" and "tails" are both equally likely.

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- Our Example: Our "coin" is biased and either "heads" or "tails" is more likely.

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- Our Example: Number of "heads".

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- **Before performing your test you must determine for which values of this test statistic you are forced to *accept* the null hypothesis and for which values of this test statistic you will *reject the Null Hypothesis* and *accept the Alternate Hypothesis*.**

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- **Before** performing your test you must determine for which values of this test statistic you are forced to *accept* the null hypothesis and for which values of this test statistic you will *reject the Null Hypothesis* and *accept the Alternate Hypothesis*.
- Our Example: Accept if $7 < \text{Number Heads} < 18$, and reject otherwise.

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- Our Example: Accept if $7 < \text{Number Heads} < 18$, and reject otherwise. Hence the critical region are the integers in $[0, 7]$ together with those in $[18, 25]$.

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- Our Example: Should be near 5 percent. Check it! .

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- In order to assure yourself that you can call your results *statistically significant* you must set your significance level to be less than 5 percent.
- In order to assure yourself that you can call your results *highly significant* you must set your significance level to be less than 1 percent.

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- Discussion: How might you approximate a type 2 error in our setting?!

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- Our Example: Assume the "die" has a 40 percent chance of coming up heads. What is the power of our test?

Is Hanover Water Yummy?

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 - ...and the protocol of how you will run the experiment. For example can you make it double blind? Are there any obvious