Read for Friday Discussion

First, in case any of you are feeling like you are lost or not quite "getting it," I would like to remind you that all we are doing is taking letters, and forming a set equal to all the words you can make with the letters. We throw in the "hats," [e.g.,$\hat{a}$] so that we have ways of removing letters [that is, we have inverses...just like subtraction lets us remove the effect of addition], and then, if we want to go from one free group to another we simply say where each letter goes.

We have spent some time discussing Free semi-groups and Free groups. We have also discussed, en route to discussing Free groups, the idea of a "finitely Generated Semigroup," which is simply a Free Semigroup (on some finite set **S**) with some relations imposed upon it.

There are other structures out there that can be thought of as Free *Groups* with some relations added. These structures are similar to semi-groups, but they have a bit more structure in that **all elements have inverses** . We will be discussing these on Friday. Specifically on Friday we will be (in groups) thinking about how we can create threse structures from Free groups by adding relations.

I would like for you to investigate the following structures so you have an idea of what they "feel like." This means you should look at the structure and create a multiplication table for it.

1. Modulo n: We have been discussing Modulo n for much of the term, and it can be modelled by taking a Free group and adding a certain relation to it. Note that we are thinking here of modulo n with its **additive** structure. So, for example, in modulo 10, 4 has the inverse 6 because 4+6=0 [in modulo 10].

2. Symmetries of a rectangle. This is the structure you saw in your homework. You take a rectangle [a non-square rectangle], and you should number the corners so you know where you started. Then, you think of all the things you can do to the rectangle that doesn't change how the rectangle looks [except that the numbers on the corners may be different]. For example, you can rotate the rectangle 180 degrees (that is, one half turn) because that will return the rectangle to what it looked like before. You cannot, however, turn the rectangle only 90 degrees because then its long side would be where its short side used to be, and vice versa. Similarly, you can flip the rectangle over its center horizontally. You can also flip it vertically. You can also do nothing. These are the only 4 things you can do if we consider any two things the same if the corners end up in the same position. That is to say, we could rotate the rectangle 3 full turns, a rotation of 1080 degrees, but its corners would be in the same place as when we started.

So this structure has four elements in it: N,R,V,H, where N="do nothing," R="Rotate 1/2 turn," V="flip vertically," and H="flip horizontally."

If you were to rotate the rectangle 1/2 turn and then flip it horizontally, you would end up in the same position as flipping the original rectangle verticlly, so we would say that F*H=V.

3. Symmetries of a square. This is the same idea as in 2, but now we have new things we can do....for example we can now rotate only 90 degrees (since this is a square and not a rectangle). We can also flip across diagonals.

This structure has 8 elements. 3 of these are rotations, 1 is "do nothing," and there are 4 flips: vertical, horizontal, and 2 diagnoals.

4. Free Abelian Group. This is just like a Free group, but we want any two words to commute. Normally, for example, (abc)*(baa)=abcbaa, but we want that to be equivalent to (baa)*(abc)=baaabc. What can we do to make sure our operation is such that X * Y=Y * X?