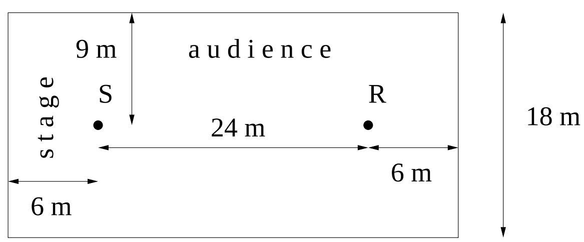
# Math 5: Music and Sound. Homework 8

due Fri May 25 ... but best if do relevant questions after each lecture

Half the usual length to give you time to work on projects!

1. Here's a musical example of diffraction of sound. A trombone's bell (where the majority of sounds radiates from) is about 17 cm in diameter.
   (a) Which frequency range do you expect to radiate roughly in all directions from the bell?
   (b) Which frequency range do you expect to emerge like a directed beam from the bell?
   (c) The trombone plays a 400 Hz note with lots of high harmonic strength. Comment on the expected difference in timbre for listeners standing along the axis of the trombone compared to those off to the side. (You may remember this from the Barton Workshop).
   (d) Compute the angular width of the beam for the $20^th$ harmonic of this note.

2. Here's a simple auditorium shown in plan view, with a performer (source S) and audience member (receiver R):




   (a) Our ears meld together echoes separated by less than about 50 ms (thousandths of a second). Use geometry to compute the time difference in ms between the arrival of direct sound and the *first* reflection off the walls. [You'll need to figure out which reflection(s) arrives first; show this on a diagram. Only consider reflections in the plane shown.]
   (b) What is the lowest pure tone frequency that would cause *destructive* interference between the direct and first reflected paths? BONUS: what is the formula for all such frequencies?

3. Here you explore how nonlinearity affects sound reproduction. Sketch the pure tone signal $y = f(t) = \sin(200\pi t)$ for $0 < t < 0.02$. We will transform the signal by composing with another function $g$ or $h$. This represents distortion by electronics such as an amplifier.
   (a) Let $g(y) = 2y$. Is $g(y)$ a linear or nonlinear function? Write the composed function $g(f(t))$ as a function of $t$ alone (by substituting in for $y$). Add $g(f(t))$ to your sketch. Has the signal shape changed?
   (b) Now let $h(y) = y + y^2$. Is $h(y)$ a linear or nonlinear function? Write $h(f(t))$ as a function of $t$ alone, and add it to your sketch. (You may want to use `fooplot.com` or some such to get the sketch right). Has the shape changed? Since $h(f(t))$ is also periodic (what is its period?) it should be able to be written as a Fourier series. Use a trigonometric identity (*e.g.* look in my math review notes) to reduce it to a weighted sum of pure sinusoids. Write down the resulting $c_j$ amplitudes. Are any frequencies present that were *not* in the original signal? How could this change the timbre?

   By the way, this type of distortion happens in tube amplifiers and is even claimed to be beneficial by the tube faithful ...