Math 5: Music and Sound. Homework 6—updated

due Fri May 11 ...but best if do relevant questions after each lecture

Don't forget to upload sounds with intelligent discussion to the Aural Postings, relevant to any course material—you are learning a lot so use your creativity!

1. Assume strumming an electric guitar excites the first 6 modes of a single string equally strongly (α_1 through α_6 are equal; as you know this is not true, but it is a useful idealization), and assume the string is 'open' (no fingers). Consider two electric pickup positions: A is 1/3 of the way along the string, whereas B is 1/12 of the way along. Use diagrams of string mode shapes (or the mode shape formula $y_n(x) = \sin(n\pi x/L)$) to estimate the strengths of the partials in the signals recorded by pickup A and pickup B. Sketch the two spectra and discuss resulting difference in timbre.

2. Guitar builders uses a 'rule of eighteen' to choose where to place each fret relative to the last one (read Sec. 3.15 of Loy book). What is the error in cents between the semitone produced by the rule of eighteen and the usual equal-tempered semitone? To get this result, what do you have to assume about string tension when the finger presses on the fret?

3. The tenor trombone is 2.75 m long in '1^{st} position' (shortest). The second air column mode is being played (B♭2); assume that in the following the player keeps in this same pipe mode (and that the bell does not cause any frequency-dependent effective length changes).[1]

   (a) How far must the slide be moved in order to go down an (equal-tempered) minor third? [Hint: draw a diagram of the trombone in both positions to understand the change in length]

   (b) How far must it again be moved to go down another minor third? Why is this different?

   (c) Congratulations; you've just worked out where 4^{th} and 7^{th} position are. Bonus: did your answer depend on which mode the player uses?

4. In the file `tube_slaps.wav` available on the homework page I recorded the sounds of two tubes A then B being slapped (the air columns excited). Use your detective skills to deduce for each one, the length, and whether the ends are open or closed. From the data could you say anything about their widths?

5. A bass flute (that we heard Jos Zwaanenburg of the Barton Workshop play in class) has a lowest note of C3.

   (a) Compute the length the flute needs to be, remembering the flute behaves like an open-open pipe.

   (b) How far from the (non-mouthpiece) end of the flute should a finger-hole be placed so that when it opens the pitch goes up by a whole tone?

6. A small empty bottle has volume 500 cm^3, and a cylindrical neck of length 5 cm and diameter 19.5 mm.

   (a) Compute the Helmholtz resonant frequency [Hint: convert everything to m, m^2, m^3 first!]

   (b) How much (cm^3) water should be poured into the bottle so that the resonant frequency changes by a perfect fifth (use 3:2)? Does it go up or down in frequency?

[1] As one of you noticed, if you ask what length of pipe has its 2nd mode at the frequency of B♭2, you get L = 2.2 m, as opposed to 2.75 m. This is a nice illustration of how the bell and the conical part of the bore changes the mode frequencies away from those of a uniform pipe. In this question I want you to assume that the extra length from the slide gets added to the 2.75 m and the effective length remains a constant fraction of the actual length. So the frequency changes in inverse proportion to the length.

(c) Going back to the empty bottle, if the half-width of frequency response is found (*e.g.* by singing) to be half a semitone (50 cents), what is the Q factor and the decay time?

(d) Compare this to the half-width of a wine glass mode with a decay time of 1 sec (assume it has the same natural frequency as the bottle). Why then is it much harder to excite a wine glass than a bottle by singing?

(e) With the empty bottle, if the amplitude of motion of the air in the neck is 5 mm (5×10^{-3} m), find the maximum speed (m/s) and maximum acceleration (m/s^2) of the air in the neck [Hint: this reviews the derivation of simple harmonic motion from Newton's 2nd Law]

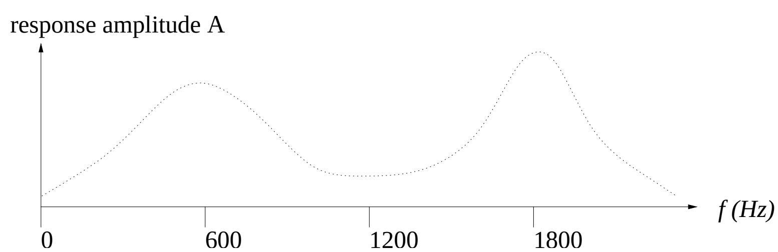
7. These will require some thought and discussion. Please try to be brief (one or two sentences) but precise.

(a) Explain why even though a flute is longer than a violin, it cannot produce a pitch as low as that of a violin.

(b) Explain why even though almost all glass bottles are shorter than a flute, when blown across they produce a pitch much *lower* than the lowest note of a flute. [Note: such bottles are also shorter than *half* the length of a flute; this eliminates one easy explanation!]

(c) When a recording of an instrument playing a note is heard in reverse, the instrument is often unrecognizable, even though the frequency spectrum must be *identical*. (Try this with the `audacity` effect `Reverse`!) What does this tell you about how we identify instruments?

8. SHORTER REPLACEMENT FOR THE ORIGINAL LAST QUESTION. The frequency repsonse (steady-state amplpitude A vs driving frequency f) of a pipe is shown below. Note that it has two peaks, each peak corresponding to a resonance of a single pipe mode (only the first two modes fall in the frequency range shown).



(a) Estimate the length of the pipe. Is it open-open or closed-open?

(b) Estimate the Q factors of the first and second resonant modes of the pipe.

(c) Estimate the half-width *in semitones* for the second resonance.