Math 5: Music and Sound. Homework 4

due Fri Apr 27 ... but best if do relevant questions after each lecture

1. In class we discussed that the Doppler frequency shift is small for everyday speeds. Here we explore that a bit more.
   (a) A siren on an ambulance traveling at 43 miles per hour[1] towards you emits a C5 note. What note will you hear?
   (b) Repeat the above except if the siren is fixed and you're driving towards it at 43 mph.
   (c) Looking at the two above answers we notice the following approximate rule for small speeds ($v \ll c$): for every 1% of the speed of sound the relative motion is (regardless of who's fixed or moving) you get a 1% change in frequency (nice and simple!) How many *cents* does a 1% change in frequency correspond to? (This question is to clarify that cents are not %)
   (d) Apply the rule to figure out if you walk towards an orchestra at 4 mph, how many cents sharp it will appear to be.
2. Alice (A) stands at the left end of a train, and Bob (B) at the right end. The train moves to the right at a constant speed of about half the speed of sound (Japanese bullet train!). Draw a spacetime diagram showing just A and B (not the train). At $t = 0$ the two people clap simultaneously (and the sound travels in the fixed air, not inside the carriages). Add their sound pulses to your diagram and use it to answer the question: do they *hear* each other's claps simultaneously? If not, who hears first and why?[2]
3. How long would it take sound to travel once around the entire Earth? (BONUS: Why don't you hear refocused sounds in the USA which were emitted in China?)
4. Download the sound `toneslouder.wav` from the website and listen to it at low volume. By using headphones and listening carefully you should be able to hear all six tones comfortably with the computer volume level fixed. (CAREFUL: it goes from very quiet to very loud so *please start at low volume until you've heard how loud it gets!*).
   (a) Use `audacity` to look at the graph of the signal and write down the approximate amplitudes of the six tones. (To see the first couple you will need to zoom vertically and stretch the whole track to be nice and tall).
   (b) Use `praat` to take the spectrum (not spectrogram!) of each of the tones in turn (please select the whole 1 second each time otherwise your results will vary). List the maximum heights of the six peaks—they are given in dB automatically by `praat`. What sequence do you notice?
   (c) By what factor is the *intensity* (not amplitude) in the last of the tones greater than the first? Express this factor in dB.
5. A clarinettist playing full volume radiates only about 0.05 W of acoustic power. Assume the sound comes out equally in all directions.
   (a) What is the intensity (power per unit area) reaching a listener at distance 2 meters?
   (b) What is this intensity in dB? (use the standard reference $I_r = 10^{-12}$ W/m^2)

[1] 1 mph = 0.45 m/s
[2] Albert Einstein used this exact argument with pulses of *light* in his world-shattering theory of Special Relativity!

(c) If the clarinettist is joined by 2 more playing equally loud but everything else stays the same, compute the intensity in dB.

(d) Going back to one clarinettist, how far away do you have to go so that they are on the threshold of human hearing (0 dB)?[3]

6. A thunderstorm clap is measured to be 130 dB when the thunderstorm is 1/5 mile away (1 mile = 1608 m).

   (a) The thunderstorm moves to 10 miles away. What intensity in dB would you expect now? [Hint: use ratios]

   (b) Compute the delay in seconds between thunder and lightning for these two distances (1/5 mile and 10 miles). Assume the light is instantaneous.

7. A mass-spring oscillator has a spring strength $k = 1000$ and mass $m = 0.001$ (let's not worry about the units).

   (a) Find the natural frequency f_0

   (b) If the spring is made 4 times stronger, what is the new natural frequency and how (by what interval) has its musical pitch changed?

   (c) Going back to the original spring, if you wanted to go *down* a perfect fifth from the original musical pitch, what value of mass m would you need?

8. The signal $e^{-t/\tau}\sin(\omega t)$ describes an oscillation decaying after it has been struck at $t = 0$. Assume $\tau = 2$ sec.

   (a) What ω is needed so that the period is $T = 0.2$ sec?

   (b) Sketch the signal, showing the *envelope*. What is the interpretation of τ?

   (c) What is the amplitude 4 seconds after being struck?

   (d) What is the Q-factor?

[3] I find this answer a little shocking. It just shows how much background noise there is, how are ears get bombarded, etc. Also relevant is the no-refocusing-from-China issue!