Math 5: Music and Sound. Homework 3

due Fri Apr 20 . . . but best if do relevant questions after each lecture

Below you may take the speed of sound to be 340 m/s.

1. review tuning systems.
   (a) The 'just' minor third is 6:5. However going up by this interval a certain number of times (how many?) is supposed to give an octave. Compute the actual ratio difference from the octave and express it as a fraction of a semitone.
   (b) Starting from a key note of C = 262 Hz, compute the frequency of the Pythagorean-tuned F in the C-major scale. Also give the frequency of this note if it were equally-tempered tuned relative to this C (careful, I did not say A was exactly 440 Hz; rather work starting at the given C).
2. The spectrum of a bell[1] contains the following main partials (in Hz): 327, 535, 710, 1169, 1751, 2348. Predict and explain with reasons what 'strike tone' apparent frequency, and corresponding musical pitch, you would hear. Would an additional partial at 2925 strengthen or weaken this impression and why?
3. When I listen to the bell sound in AD/DC's "Hells Bells" (Audio Posting by JB Cholnoky), I hear a strong impression of A3 (220 Hz), and I think you will too[2]. However there is *no partial* at 220 Hz or anywhere very near it. Please explain this (you will want to our sound analysis tools).
4. Compute how the Helmholtz theory of dissonance would rate the consonance or dissonance of the following intervals, counting only the partials up to 2000 Hz. Assume the lower note is at 300 Hz in each case, as in the worksheet. Here's a scoring system that's a bit more realistic than the one in class: count 1 for a fully dissonant pair of partials (less than 10% different), and 0.5 for a marginally dissonant pair (between 10% and 15% different). Let's say partials closer than 15 Hz (the max beat frequency) don't count as dissonant at all. Please state how many fully and slightly dissonant partials there are in each case.
   (a) the minor 6th (use just tuning with 5:8)
   (b) the perfect 4th (use just tuning)
   (c) the minor 2nd, *i.e.* one semitone (use equal-tempered tuning)

   Discuss which interval is most dissonant, and which the least, according to the theory.
5. Compute the wavelength of a pure sinusoid signal traveling through air at the following frequencies
   (a) the musical pitch C6 (a very high soprano note)
   (b) 15,000 Hz (upper range of hearing)
   (c) 20 Hz (lower range of hearing)
6. Assume the human ears are about 0.2 m apart, and that the head poses no obstruction to the propagation of sound. Let's imagine sound waves arrive from your right (*i.e.* 90 degrees to the right of straight ahead).

[1] I took this example by tweaking one of the partials from the 'Mears 1850' bell you can hear at `http://www.hibberts.co.uk/ears.htm`

[2] There's also a low E3 but I find it weaker

(a) Compute the time delay between a signal arriving at the left and right ears.

(b) If the signal arriving at the left is $\sin(880\pi t)$, compute the *phase* at the right ear, *i.e.* the value ϕ in in right ear signal $\sin(880\pi t + \phi)$

7. Here are two echoes I recorded on or near campus. Analyse them with `audacity` in order to answer the questions. [Hint: you will want to zoom in vertically by clicking on the y-axis until you can measure accurately what you want]

   (a) 'Mystery echo A': I was standing in front of a wall which gave the strongest reflection (ignore the other weaker ones). How far away was I?

   (b) 'Mystery echo B': I stamped my foot a few times (the last one was best). What arrangement of walls might be causing the echo? Please compute relevant distance(s).

8. Imagine a musician was playing an A4 (440 Hz). What frequency, and musical pitch, would you hear in the following (admittedly unlikely) situations?

   (a) You're on the ground while the musician is in a (silent!) jet plane moving at 170 m/s (380 mph) towards you.

   (b) The same except moving away from you.

   (c) The musician is on the ground and you're in the plane moving at 170 m/s towards the musician.

   Bonus: If the musician played actual music, would the *speed* (tempo) of the music change as well as the pitch?

9. Analyse the sound 'Moving train whistle' I recorded of a train blowing its whistle while coming towards me, then away from me (I was at rest relative to the ground). Deduce how fast the train was moving, in meters per second, relative to the ground. [Hint: I recommend you convert to WAV then use the `praat` spectrogram with 0.2 s window and 0-1000 Hz range, then measure and compare carefully one or two partials of the first vs last blasts].

   Bonus: you may notice some of the partials of the last blast are present more quietly in the first—what do you think might be causing this?