Math 5: Music and Sound. Homework 1

due Fri Apr 6 ... but best if do relevant questions after each lecture

First install audacity onto your laptop and make sure you can record a sound, play it back. See our Software page for help. Please do this before Tues X-hour, since then you can bring your laptop and try it out in class.

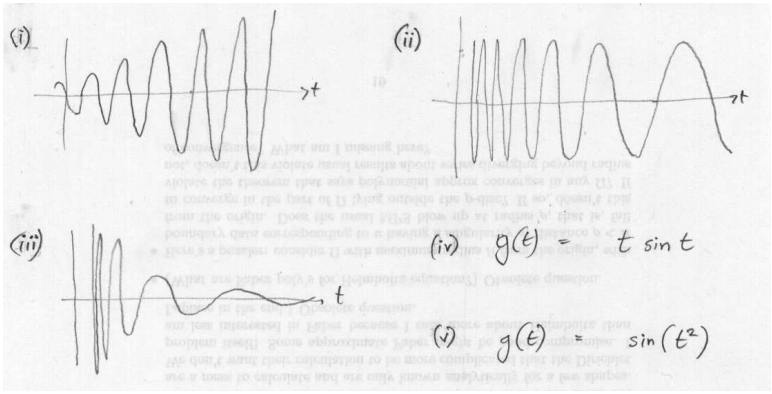
1. (a) What is the period, and frequency, of the signal frac($100t$) ? (you may assume t is measured in seconds, and frac means fractional part)
   (b) What is the period, and frequency, of the signal $\sin(100t)$?
   (c) Compare the frequency ratio 3:2 to that given by 7 equal-tempered semitones. Express their difference both as a *percentage* error (*e.g.* 999 Hz is 0.1% flat relative to 1000 Hz), and in *cents*.

2. Given the modern A4 of 440 Hz, compute the frequencies of the following notes using equal-tempered tuning: C4 (careful: this is the first C *below* A4), A1, D#3, F6 (the soprano top F in Mozart's *The Magic Flute*).

3. Find which (equal-tempered, modern) note names the following frequencies are nearest, and express how out of tune they are from these notes, in cents.
   (a) Handel's 'A4' tuning fork from 18th century which still survives and is tuned to 422.5 Hz (no, it didn't drift).
   (b) 256 Hz (which was proposed by scientists in 1939 as a standard for C4 [why, do you think?], and rejected, thankfully)
   (c) The 'interference' hum you sometimes hear at 60 Hz due to our AC electrical system.

4. Download *Mystery Sound 1* from the HW page.
   (a) Find the (couple of main) component frequencies as accurately as you can using `audacity`. [Hint: make spectrum fill the screen, use Log and the longest transform length 16384. `audacity` suggests note names but you have no idea how far off they are].
   (b) Express them as notes in the equal-tempered system with tuning errors from these notes in cents.
   (c) Roughly what musical interval do these frequencies produce? The interval is close to one involving small integers—which ones?
   (d) BONUS: Do some detective work and explain what this familiar sound is and why it is not as familiar as usual...

5. Five functions, which we can interpret as pressure vs time, are given below either as a graph or a formula. State which (and there may be none, or more than one) of them are...
   (a) Decreasing in amplitude and in frequency
   (b) Getting louder but at constant pitch (ignore any subtle pitch-loudness perception issues for now!)
   (c) Increasing in pitch but not in amplitude
   (d) Decreasing amplitude but at constant frequency



6. The pressure signal $\sin(880\pi t + \pi/3)$ can be written as $A\sin(880\pi t) + B\cos(880\pi t)$. Find the constants A and B. Explain with reasons whether you expect this signal to sound the same as $\sin(880\pi t)$ to the ear.

7. Two sinusoidal tones of frequencies 400 Hz and 402 Hz are played together both with amplitude 1.

   (a) Write down a formula for the combined (added) signal.

   (b) Describe what you would hear. [you might check this by generating then mixing two tone tracks with `audacity` but this is not required].

   (c) Use a trig identity to re-express this signal in a form more useful for *understanding* what you hear. Explain how the two terms in this formula correspond to aspects of what you hear.

8. Draw a time axis labelled 0, 0.01, 0.02 etc up to 0.08 (think of this as measured in seconds).

   (a) Sketch (without using a computer, since you'll want to practise for doing this in an exam) the graphs of $\sin(100\pi t)$ and $\sin(\frac{200}{3}\pi t)$ on these axes. [Hint: first get the zero-crossings right]

   (b) What are the individual frequencies, and periods, of these functions?

   (c) Add to your sketch (in a different color) an estimate of the *sum* of the functions.

   (d) What is the period of the sum? What musical interval would you hear when the signals are played together?

   (e) What would the period of the sum become if the second frequency was changed very slightly to 33 Hz?

9. Upload to our Aural Postings page an interesting sound (maybe recorded with audacity and saved in OGG format) illustrating a concept from the first week of class (explain *how* it illustrates it!)