Partial Differential Equations

Math 53 Fall '19
Instructor: Yoonsang Lee
Office Location: 206 Kemeny
Class Meetings: MWF 11:30 - 12:35

Course Description

Partial differential equations play critical roles in wide areas of mathematics, science and engineering. This is an introductory course, and is accessible to undergraduate and graduate students in mathematics and other scientific disciplines who have completed the prerequisites. Examples will come from both linear and non-linear partial differential equations, including the wave equation, diffusion, boundary value problems, conservation laws, and the Monge-Ampere equations. The focus will be on learning the analytical tools needed to solve these problems. This course is offered in Fall term every other year.

Prerequisites

MATH 22 (24), MATH 23 or instructor approval.

Course Learning Outcomes

In this course, students will

- Learn analytic tools to solve Laplace, Poisson, Heat, and Wave equations along with nonlinear equations.
- Be introduced to abstract function spaces such as Hilbert spaces, which motivates further studies in advanced mathematics such as functional analysis,geometry, and applied and computational mathematics.

Teaching Methods & Philosophy

The course will follow the standard lecture-style teaching complemented with computer simulation, visualization, etc. to introduce real applications of the course materials in science and engineering and to motivate students to learn advanced mathematics. Instead of introducing abstract ideas and then applying them to applications, the class will develop ideas from several examples and apply to other applications.

Texts & Materials

The main textbook is

- W. A. Strauss, *Partial Differential Equations: An Introduction*

Other references include

1. J. David Logan, *Applied Mathematics (Chapters 6-8)*
2. L. C. Evans, *Partial Differential Equations*
3. V. I. Arnold, *Lectures on Partial Differential Equations*

Assessment & Grading

The homework and tests include problems to measure quantitative (calculation-style) and qualitative (proof-style) understanding of the materials. The grading will base on the following contributions:

1. Six homework assignments 30%,
2. Midterm 30%,
3. Final 40%.

Final exam times and locations are determined by the registrar. The final exam is two hours long, and final course grades will be curved. Neither the midterm nor the final examination will be given early unless there are extenuating circumstances. For the midterm and final there is a seven day statue of limitations for grading appeals, and the seven day period begins on the day that an exam is returned.

Students are responsible for understanding the academic integrity rules in place at Dartmouth. Explanations of Dartmouth's Academic Honor Principle can be found at
https://students.dartmouth.edu/judicial-affairs/policy/academic-honor-principle,
and details about citing sources are available at
http://writing-speech.dartmouth.edu/learning/materials/sources-and-citations-dartmouth.
Ignorance of the Academic Honor Principle will not be considered a mitigating excuse if a violation occurs.

Dartmouth Policies

Student Accessibility and Accommodations

Students with disabilities who may need disability-related academic adjustments and services for this course are encouraged to see me privately as early in the term as possible. Students requiring disability-related academic adjustments and services must consult the Student Accessibility Services office in Carson Hall 125 (phone: 646-9900 or Student.Accessibility.Services@Dartmouth.edu).

Once SAS has authorized services, students must show the originally signed SAS Services and Consent Form and/or a letter on SAS letterhead to me. As a first step, if you have questions about whether you qualify to receive academic adjustments and services, you should contact the SAS office. All inquiries and discussions will remain confidential.

Religious Observances

Some students may wish to take part in religious observances that occur during this academic term. If you have a religious observance that conflicts with your participation in the course, please meet with me before the end of the second week of the term to discuss appropriate accommodations.

Course Schedule and Topics

Week 1: Introduction

- Day 1: PDE modeling
- Day 2: Review of ODEs
- Day 3: Series solutions of ODEs

Week 2: Classification of PDEs

- Day 1: First-order linear equations
- Day 2: Initial and boundary conditions, well-posed problems
- Day 3: Types of second-order equations

Week 3: Wave Equations

- Day 1: Vibrations of a drum
- Day 2: Causality and energy
- Day 3: Reflections of waves

Week 4: Diffusion Equations

- Day 1: Diffusion on the whole Line
- Day 2: Diffusion with a source
- Day 3: Fokker-Plack equations

Week 5: Boundary Value Problems

- Day 1: Separation of Variables, boundary conditions
- Day 2: Fourier transforms - orthogonality and completeness
- Day 3: Laplace's equation, Poisson's equation

Week 6: Eigenvalue Problems

- Day 1: Computation of eigenvalues
- Day 2: Spectral theory

- Day 3: Asymptotics of eigenvalues

Week 7: Distributions and Weak Formulation

- Day 1: Green's functions
- Day 2: Distributions
- Day 3: Weak solutions

Week 8: Function Spaces

- Day 1: Hilbert space
- Day 2: Lax-Milgram theorem
- Day 3: Banach space

Week 9: Abstract Formulation

- Day 1: Second-order elliptic equations
- Day 2: Second-order linear evolution equations
- Day 3: Semigroup theory

Week 10: Nonlinear PDEs

- Day 1: Conservation laws
- Day 2: Calculus of variations
- Day 3: Fully-nonlinear equations: Monge-Ampere equations