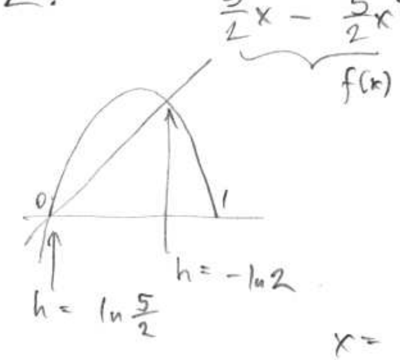
Barnett 10/14/05

T3.2: $\underbrace{\frac{7}{2}x - \frac{5}{2}x^2}_{f(x)} = x$ so $x = 0$ or $3 = 5x$, $x = \frac{3}{5} = p$ f.p.



$f'(x) = \frac{5}{2} - 5x$ $f'(p) = -1/2$.

$h = -\ln 2$.

All of $(0,1)$ sucked into p. $\Rightarrow h = -\ln 2$.

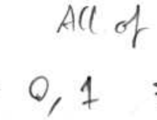
$x = 0, 1$ EP to source at $x=0$ $\Rightarrow h = \ln 5/2$.

Also, officially $x = 1/2$ & all its preimages have h undefined. (set of measure 0).

3.1. a) $a - x^2 = x$ so $x^2 + x - a = 0$ $p = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + a}$

$a_1 = -1/4$.

b) $x_n \to -\infty$



note $f(x) < x$ $\forall x$

c) $f'(x) = -2x$ so want $|2p| < 1$ ie $|1 - \sqrt{1+4a}| < 1$

so $\sqrt{1+4a} < 2$, ie $a < 3/4$ $a_2 = 3/4$

d) $f^2(x) = a - (a-x^2)^2 = x$ so $x^4 - 2ax^2 + x - a + a^2 = 0$

but factor out $(x^2 + x - a)$: $(x^2 + x - a)\underbrace{(x^2 - x + 1 - a)}_{\text{must be p-2 roots}} = 0$
↖ 2 period-1 fp.

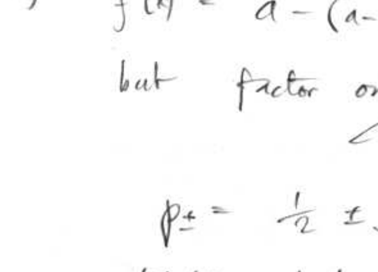
$p_\pm = \frac{1}{2} \pm \sqrt{\frac{1}{4} + a - 1}$

stability $\underbrace{|f'(p_-) f'(p_+)|}_{} < 1$ → $|4p_-p_+| = 4|1-a| < 1$

notice product of roots $= 1 - a$ (const term in quadratic)

p-2 sink for $(3/4, 5/4) \ni a$

e)



← a=2 case.

Conjugate to $G(y) = 4y(1-y)$ via $x = -2 + 4y$

Check it!

$\Rightarrow$ chaotic, since G was proven to be so.

3.4: All orbits are either: $x \in (0,1)$ AP to f.p. w/ $h = \ln 1/2 < 0$.

$x = 0$ or 1, $h = \ln 5/2 > 0$ but AP $\Rightarrow$ not chaotic

$x < 0$ or $x > 1$: unbounded $\Rightarrow$ not chaotic.

See defn. of chaotic, p.110

3.7: Need $\forall x_0 \in [0,1]$, $\forall \varepsilon > 0$, there's a periodic pt p st. $|p - x_0| < \varepsilon$. & use Fixed Pt Thm

Let $k > \log_2 \frac{1}{\varepsilon}$, let J = 1st k letters of x_0's itinerary $= S_1 \cdots S_k$. Then $f^k(S_1 \cdots S_k S_1) \supseteq S_1$