MATH 53 — HW1

Barnett 10/2/09

T1.8 solution.

Please read carefully to learn how to write a clear proof.

Proof:

G^k has 2^k fixed points. (we trust the book & worksheet on this one; I have yet to see a rigorous proof of this.)

f.p. of G^k associated with lower periods ($n<k$), let's call this N:

$$N = \sum_{\substack{n=\text{divisor}\\ \text{of } k}} n \cdot (\#\text{ period-}n\text{ orbits}) \le \sum_{\substack{n=\text{divisor}\\ \text{of } k}} (\#\text{ f.p. of } G^n)$$

since # f.p. assoc with p-n. bounded by # f.p. of G^n.

$$\le \sum_{n=1}^{k-1} (\#\text{ f.p. of } G^n)$$

$$= \sum_{n=1}^{k-1} 2^n = 2^k - 2$$ by geometric series.

f.p. of G^k assoc. w/ period-k $= 2^k - N$ exactly

$\geq 2^k - (2^k - 2)$ using above upper bound on N

$= 2$

So there are at least 2 f.p. of G^k assoc w/ period-k, so there exists a period-k orbit. $\square$

Notice:

- we wrote equations (<u>equalities</u>), then used logic to replace with upper / lower bounds for things. (inequalities).
- We never said, "Suppose every $n=1\dots k-1$ is a divisor of k", which is obviously rubbish for $k>2$.
- We were careful to explain it is <u>period-n</u> orbits which account for f.p. of G^k, not merely f.p.'s of G^n.
- we defined a new symbol N to break into two simpler steps.