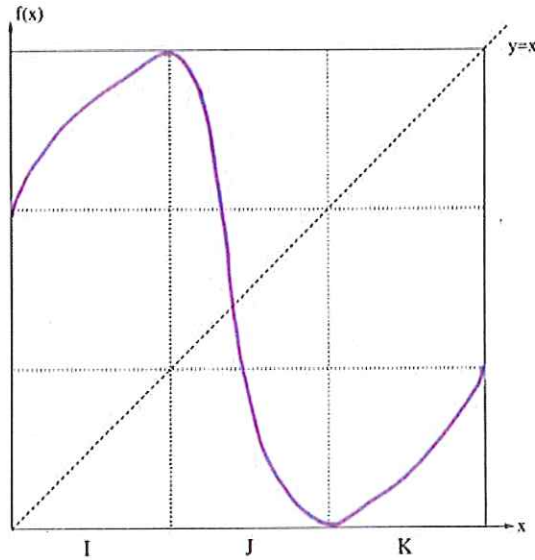
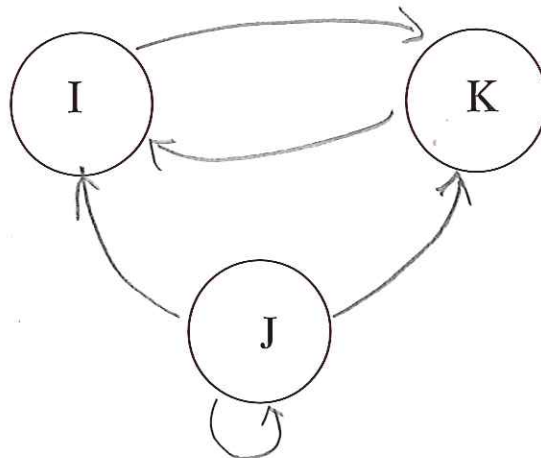


Worksheet #9: Transition graphs



(1) Draw the transition plot of the graph above. Hint: Is $I \subset f(I)$? Is $K \subset f(I)$? etc.
 No Yes



(2) Prove there is a fixed point of f in J .

$f(J) \supset J$ so by the fixed pt thm \exists a fixed pt.

(3) Prove there is a fixed point of f^2 with $p_1 \in I$ and $p_2 \in K$.

$f^2(I) = f(f(I)) = f(K) \supset I$
 so by fixed pt thm \exists a fixed pt of $f^2(I)$ in I .

(4) Categorize all possible infinite sequences of symbols. For example, is \overline{KI} legal? What if you start in J ?

$$\overline{J}$$

$$J^n \overline{KI} \quad n=0, 1, 2, \dots$$

$$J^n \overline{IK} \quad n=0, 1, 2, \dots$$

(5) Prove that periodic orbits of f have period 1 or 2 but no others.

* Now, in the case of f , we have $f^2(x) = x$ for all x .
 (This means that f is an involution.)
 Now, let x be a point in a periodic orbit of f .
 Then $f^n(x) = x$ for some $n > 0$.
 We want to show that $n=1$ or $n=2$.
 Suppose $n > 2$. Then $f^n(x) = x$ implies $f(x) = x$ or $f(x) = f^3(x)$.
 If $f(x) = x$, then x is a fixed point, so $n=1$.
 If $f(x) = f^3(x)$, then $f^2(x) = x$, so $n=2$.
 Therefore, the only possible periods are 1 and 2.

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 Therefore, the only possible periods are 1 and 2.

- There are no odd periodic orbits since IST where S is an odd numbered collection of symbols.
- only other possible way to have an odd orbit is if $x \in J$ st $f^{2k+1}(x) \in J$

$f^{2k+1}(x)$ is a monotonic decreasing function.

so $P_2 > P_1 \Rightarrow f^{2k+1}(P_2) < f^{2k+1}(P_1)$
 Assume $\{P_1, \dots, P_{2k+1}\}$ is a periodic orbit.
 Then $f^{2k+1}(P_j) = P_j \quad j=1, \dots, 2k+1$
 but this is a contradiction to f^{2k+1} being decreasing.

- There are no even periodic orbits. Why?
 If such orbit exists it must lie in $I \cup K$ since f is strictly decreasing in J .

In $I \cup K$ f is monotonic increasing
 $\Rightarrow f^2(x)$ is monotonic increasing.

Assume \exists a $2k$ -periodic $\{P_1, \dots, P_{2k}\}$
 where $k > 1$.

1st show for $k=2$. $\{P_1, P_2, P_3, P_4\}$
 If $P_1 \in I$, $f^2(P_1) = P_3 \in I$ & $f^2(P_3) = P_1$
 $f^2(x)$ is increasing $\Rightarrow P_1 < P_3$

Thus it is impossible for $f^2(P_3)$ to be P_1
 \therefore contradiction.

This movement of pts happens for $k > 2$ as well.

Thus it is impossible for there to be a k -periodic orbit for $k > 1$.