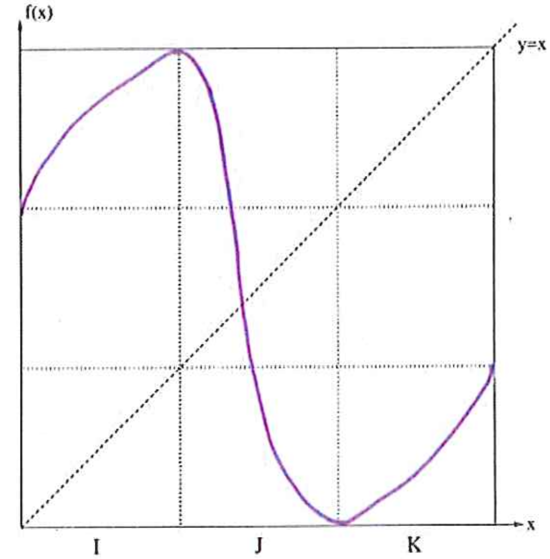
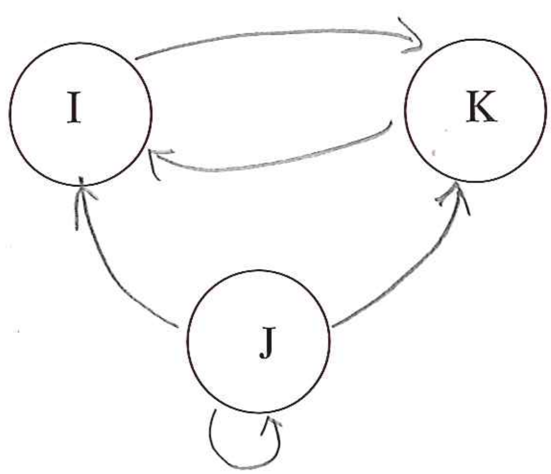
Worksheet #9: Transition graphs



(1) Draw the trasition plot of the graph above. Hint: Is $I \subset f(I)$? Is $K \subset f(I)$? etc.

No   yes.



(2) Prove there is a fixed point of f in J.

$f(J) \supset J$ so by the fixed pt thm $\exists$ a fixed pt.

(3) Prove there is a fixed point of f^2 with $p_1 \in I$ and $p_2 \in K$.

$f^2(I) = f(f(I)) = f(K) \supset I$

so by fixed pt thm $\exists$ a fixed pt of $f^2(I)$ in I.

(4) Catergorize *all* possible infinite sequences of symbols. For example, is $\overline{KI}$ legal? What if you start in J?

$\overline{J}$

$J^n\,\overline{KI}$ $\quad n = 0, 1, 2, \ldots$

$J^n\,\overline{IK}$ $\quad n = 0, 1, 2, \ldots$

(5) Prove that periodic orbits of f have period 1 or 2 but no others.

- There are no odd periodic orbits since $I S I$ where S is an odd numbered collection of symbols.

- only other possible way to have an odd orbit is if $x \in J$ st $f^{2k+1}(x) \subseteq J$

$f^{2k+1}(x)$ is a monotonic decreasing function.
so $P_2 > P_1 \Rightarrow f^{2k+1}(P_2) < f^{2k+1}$
Assume $\{P_1, \dots, P_{2k+1}\}$ is a periodic orbit.
Then $f^{2k+1}(P_j) = P_j \quad j = 1, \dots, 2k+1$
but this is a contradiction to f^{2k+1} being decreasing.

- There are no even periodic orbits. Why?
If such orbit exists it must lie in $I \cup K$ since f is strictly decreasing in J.

In $I \cup K$ f is monotonic increasing
$\Rightarrow f^2(x)$ is monotonic increasing.

Assume $\exists$ a $2k$-periodic. $\{P_1, \dots, P_{2k}\}$
where $k > 1$..
1st show for $k=2$. $\{P_1, P_2, P_3, P_4\}$
If $P_1 \in I$, $f^2(P_1) = P_3 \in I$ & $f^2(P_3) = P_1$
$f^2(x)$ is increasing $\Rightarrow P_1 < P_3$
Thus it is impossible for $f^2(P_3)$ to be P_1
∴ contradiction.
This movement of pts happens for $k > 2$. as well.

Thus it is impossible for there to be a k-periodic orbit for $k > 1$.