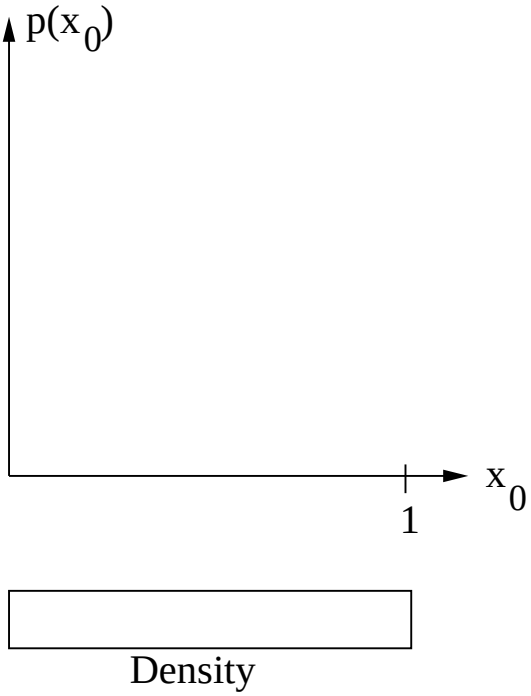
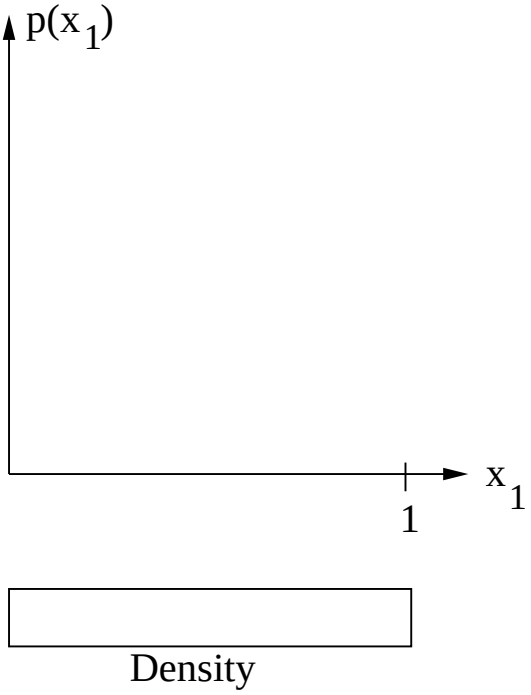
**Worksheet #10: Fractals from probablistic games**

**Part 1** Apply $f_1(x) = \frac{x}{3}$ or $f_2(x) = \frac{x+2}{3}$ with equal probability of 1/2 on each iteration.

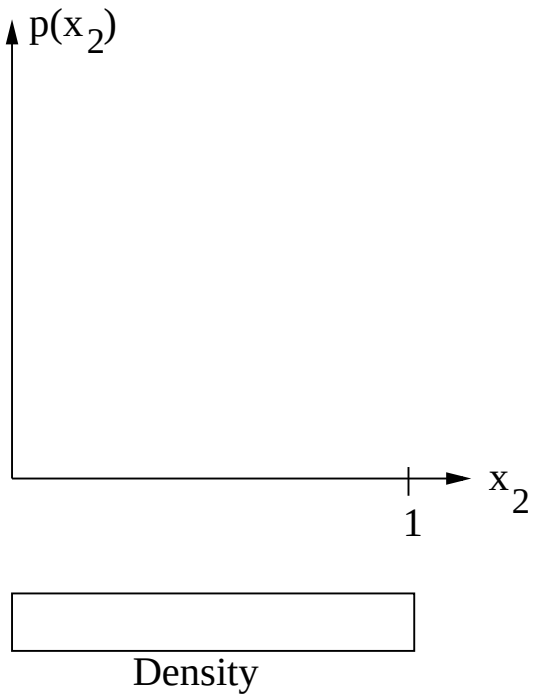
Starting with $p(x_0)$ uniform on $[0, 1]$,




Find $p(x_1)$ and sketch. [Hint: what geometrically does $f_2$ do?]



Find $p(x_2)$ and sketch.




What is $p(x_n)$?

What is the limiting attractor set as $n \to \infty$?

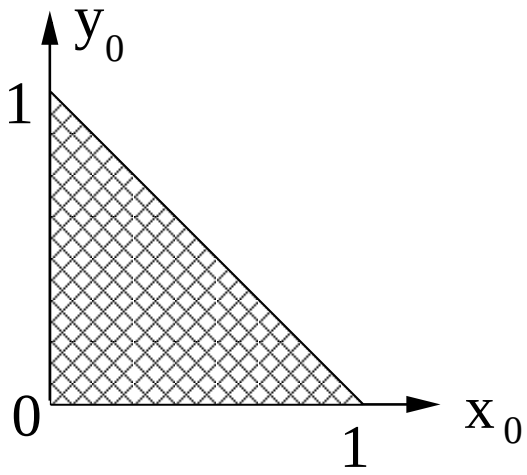
Prove an upper bound on the distance of $x_n$ to this set. [Hint: The distance is bounded by the distance from $x_0$ to the set.]

**Part 2** Now try a 2D example. Start with $\boldsymbol{x}_0$ uniform in a triangle.
Apply

$$\begin{cases} f_1(\boldsymbol{x}) &= \left(\frac{x}{2}, \frac{y}{2}\right) \\ f_2(\boldsymbol{x}) &= \left(\frac{x+1}{2}, \frac{y}{2}\right) \\ f_3(\boldsymbol{x}) &= \left(\frac{x}{2}, \frac{y+1}{2}\right) \end{cases}$$

with probabilities 1/3. Deduce the attractor set.

Uniform




Iterated