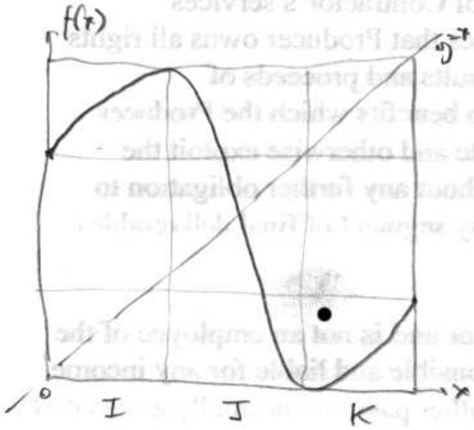
MATH 53 WORKSHEET : Transition graphs. 10/24/07 Barnett

Consider the following map



J

I

K

Draw the transition graph above. Hint: does $f(I) \supset I$?

$f(I) \supset K$? etc...

Prove there's a fixed point of f in J :

Prove there's a fixed pt of f^2 with $p_1 \in I$, $p_2 \in K$.

Categorize all possible infinite sequences of symbols:

eg is $\overline{KI}$ legal ? What if start with J ?

Prove that the periodic orbits of f have period 1 or 2, no others:

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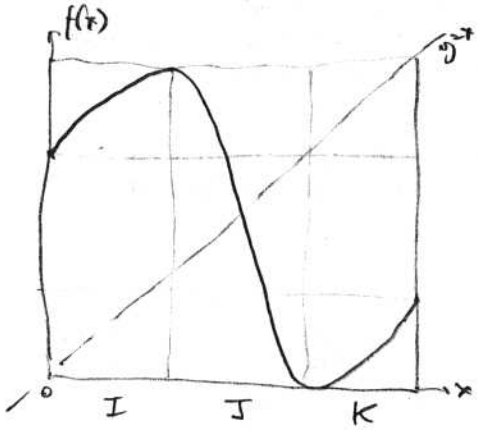
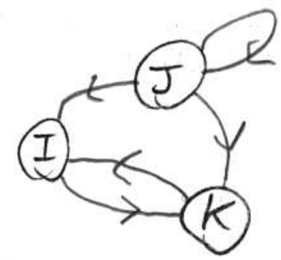
~ SOLUTIONS ~

Consider the following map [see T3.11]





Note: $J \to I$ means $f(J) \supset I$, not that there is no such $x \in J$ with $f(x) \in I$!

Draw the transition graph above. Hint: does $f(I) \supset I$? ×
$f(I) \supset K$? ✓ etc...

Prove there's a fixed point of f in J: • $f(J) \supset J$ so by Fixed Pt Thm, $\exists$ fixed pt. in J.

Prove there's a fixed pt of f^2 with $p_1 \in I$, $p_2 \in K$:

$$f^2(I) = f(f(I)) = f(K \cup \{\text{possible other intervals}\}) \supset I \quad \text{So}$$

Proof: apply fixed pt Thm to f^2 on I.

Categorize all possible infinite sequences of symbols:
eg is $\overline{KI}$ legal? What if start with J?

$\overline{J}$

$J^n\,\overline{KI}$ $n = 0, 1, \dots$

$J^n\,\overline{IK}$ $n = 0, 1, \dots$

tricky since require monotonicity of f in I, J, K intervals.

Prove that the periodic orbits of f have period 1 or 2, no others:

no odd POs >1 since: i) they cannot be in I or K because $I\underbrace{xx}_{}I$ is not possible (even number of symbols.)

ii) In J, f^{2k+1} is monotonic decreasing $(k = 1, 2, \cdots)$ so $p_2 > p_1 \Rightarrow f^{2k+1}(p_2) < f^{2k+1}(p_1)$

Applying to the PO $\{p_1, \cdots p_{2k+1}\}$ leads to a contradiction since $f^{2k+1}(p_j) = p_j \quad \forall j = 1 \cdots 2k+1$

no even POs > 2 since f monot. incr. in both I & K, so f^2 monot. incr. in both I & K,

so if $\{p_1, \cdots p_k\}$ is period-$2k$ then $f^2\{p_1 \cdots p_k\}$ preserves the ordering $\Rightarrow$ contradiction since f^2 must permute