Math 53 Chaos!: Homework 3

due Fri Oct 19 ... but best if do relevant questions after each lecture

Although it looks like lots of questions this week, most of them are pretty fast (I believe...)

2.3 (easy). Are the fixed points hyperbolic?

A. Find the slight subtlety in the proof that AB and BA always have the same eigenvalues, which underlies the lovely fact that stability is the same whichever point in a periodic orbit you pick (Remark 2.14). Specifically: write the relation stating λ is an eigenvalue of AB. Left-multiply by B and interpret this as an eigenvalue relation for BA. Are the corresponding eigenvectors the same? This argument fails for one case of λ: explain why, then use the characteristic equation to prove it in this case.

T2.7 a,b only.

2.8

Compu. Expt. 2.2: Here you can take the guts of the `explormap2d.m` code and wrap it with something to do a bifurcation diagram as requested. This is not hard but will be good programming experience building on what you already know. Print out your x-coordinate diagram for $b = -0.3$ and $0 \leq a \leq 2.2$.

T2.8 (easy)

T2.10 Give two vectors parallel to the axes. Explain the surprising result that even though one of the eigenvalues of AA^T *exceeds* 1 in absolute value, the ellipses $A^n N$ shrink to the origin.

2.9 Show a sketch as in Fig. 2.29 showing the action of the inverse cat map.

Challenge 2: Glancing at Fig. 2.31 you see this linear map has complex behavior which makes it fun to investigate. Make sure you're happy up to Step 5. Do Step 7 too on your own (darn Fibonacci again!). Then write up:

Step 6 (easy)

Step 8: plotting the solutions in the unit square will help you count them.

Step 9. (I found a simpler formula than theirs—can you?)

Step 11. Write out table only to $k = 6$ (you don't need Step 10), and treat the proof that all periods exist only as an optional BONUS.

Compu. Expt. 3.1: You can combine bits of code from HW1 and from Compu. Expt. 2.2 above, to make this Lyapunov-exponent-vs-a plot. Use fine steps in a, *e.g.* 10^{-3}, to see the jagged quality. Only once you're happy with your plot, compare to p. 237.

Hints: look at the `hw1_iter_sol.m` code I provided on the HW page. You notice it plots the difference of two nearby orbits on a *log* scale. If you take the ln of this difference, the slope of the resulting graph is literally h, the Lyapunov exponent (as explained p. 107). So you could measure the slope of this graph using eg 25 its (but not too many otherwise it stops growing). Since h can be negative, I suggest you don't start at 10^{-15} difference (since it could get smaller but you'd not be able to see this due to round-off error). Instead, why not choose a number somewhere between this and 1 ('between' in what sense?) so that you can detect + or − exponents.

A better alternative is to use only one orbit x_k but to keep track of the product $g'(x_1)g'(x_2)\ \ldots$ for that orbit. For each a, use Defn 3.1 to estimate the exponent. This method allows you to go for more than 25 its (why?)

Postponed to HW4:

T3.2 (easy and has some review of Ch. 1. Be sure to find Lyapunov *exponent* not *number*)