3/1/06

Math 50: Practise questions on post-Midterm 2 material.

A) Given random numbers uniformly distributed in [0, 1], how would you generate random samples from the pdf $f_Y(y;\theta) = \frac{2y}{\theta^2}$, $0 \le y \le \theta$?

B) If $Y \sim N(\mu, \sigma^2)$ and $Z = e^Y$, find a formula for $f_Z(z)$. This is called a 'log-normal' pdf and is important in applications. [Hint: make sure your expression is in terms of z only, not y!]

C) A small class has midterm grades 75, 91, 69, 82

i) Test the hypothesis $H_1: \mu \neq 85$ vs $H_0: \mu = 85$

ii) Test hypothesis $H_1: \sigma > 4$ vs $H_0: \sigma = 4$

} assuming underlying normal pdf.

9.4.1 binomial 2-sample

9.2.5 t-test. 2-sample

7.4.7 conf. int. via t.

7.4.21 hyp. t-test. on μ

7.5.15 χ^2 test on σ^2. (data need not be typed in).

D) Find ρ, correlation coefficient, for $f_{X,Y}(x,y) = \begin{cases} 2 & 0<y<x,\ 0<x<1 \\ 0 & \text{otherwise.} \end{cases}$

Practise post-Midterm 2 answers.

Barnett
3/3/06

A) cdf $F_Y(y) = \begin{cases} 0 & y<0 \\ \frac{y^2}{\theta^2} & 0<y<\theta \\ 1 & y>\theta \end{cases}$

inverse func solve $F = \frac{y^2}{\theta^2}$ for y $\rightarrow$ $y = \theta\sqrt{F}$

Therefore feed your unif on $[0,1]$ random #s into function $g(x) = \theta\sqrt{x}$

B) $z = e^y$ is monotonic; $A(z) = y = \ln z$, $\frac{d}{dz}A^{-1} = \frac{dy}{dz} = \frac{1}{z}$

rule is $f_Z(z) = \frac{d}{dz}A^{-1}(z) \cdot f_Y(A^{-1}(z)) = \frac{1}{z} \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(\ln z - \mu)^2}$

note sub. $y = \ln z$.

C) i) $\bar{y} = 79.25$ $\quad s^2 = \frac{1}{n-1}\sum_{i=1}^{4}(y_i - \bar{y})^2 = 89.6$

$t = \frac{\bar{y} - \overset{\mu_0}{85}}{s/\sqrt{n}} = -1.22$ (see §7.4)

but $t_{\alpha/2,3} = -3.18$ so we cannot reject H_0 at 95% conf. level.

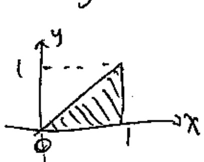
$\alpha = 0.05$, 2-sided

p-value is in fact $0.31 > 0.05$.

ii) (see §7.5) Thm. 7.5.2 $\quad \chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{3\cdot(89.6)}{4^2} = 1.68$

compare against $\chi^2_{1-\alpha,n-1} = 7.81$, can't reject H_0 here either!

D) $f_X(x) = \int_0^x 2\,dy = 2x$ so $\mu_X = \frac{2}{3}$, $f_Y(y) = \int_y^1 2\,dx = 2(1-y)$ so $\mu_Y = 1/3$



$\sigma_X = \sqrt{\int_0^1 2x \cdot x^2 dx - (2/3)^2} = \sqrt{\frac{1}{2} - \frac{4}{9}} = \sqrt{1/18}$, same for σ_Y.

$Cov(X,Y) = E(XY) - \mu_X\mu_Y = 2\int_0^1 \underbrace{\int_0^x xy\,dy}_{x^3/2}\,dx - 2/9 = \frac{9-8}{36} = \frac{1}{36}$.

$\rho = Cov(X,Y)/\sigma_X\sigma_Y = \frac{1}{36}/\frac{1}{18} = \underline{1/2}$.