Good Wolf Hunting

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TOPIC

BACKGROUND

In 1995 wolves were reintroduced to Yellowstone. This triggered a trophic cascade that increased populations of beavers and other small herbivores, led to taller trees and even changed the course of some rivers. As predator populations continue to decline across the globe it is imperative that we understand the positive environmental/ecological impact that they can have.

In this project we seek to model the effects seen in Yellowstone, and analyze population stability within ecological networks to determine if predator introduction is a viable option in other environments. We do so by creating a closed-toy ecosystem where we can study some of the underlying dynamics and hopefully garner some insight into how predator species, sometimes in small quantities, can have drastic impacts in correcting their ecosystems back to a sustainable stable point or cycle.

To construct our toy model, we include grass as the producer, elk and rabbits as the small and large herbivores to feed on the grass and grow rapidly in the absence of a predator, and wolves as the predator species sustained by eating elk and rabbits. In our model, we hope to analyze the systems before and after the introduction of a wolf population in and to compare the system dynamics for the two initial conditions.

ASSUMPTIONS

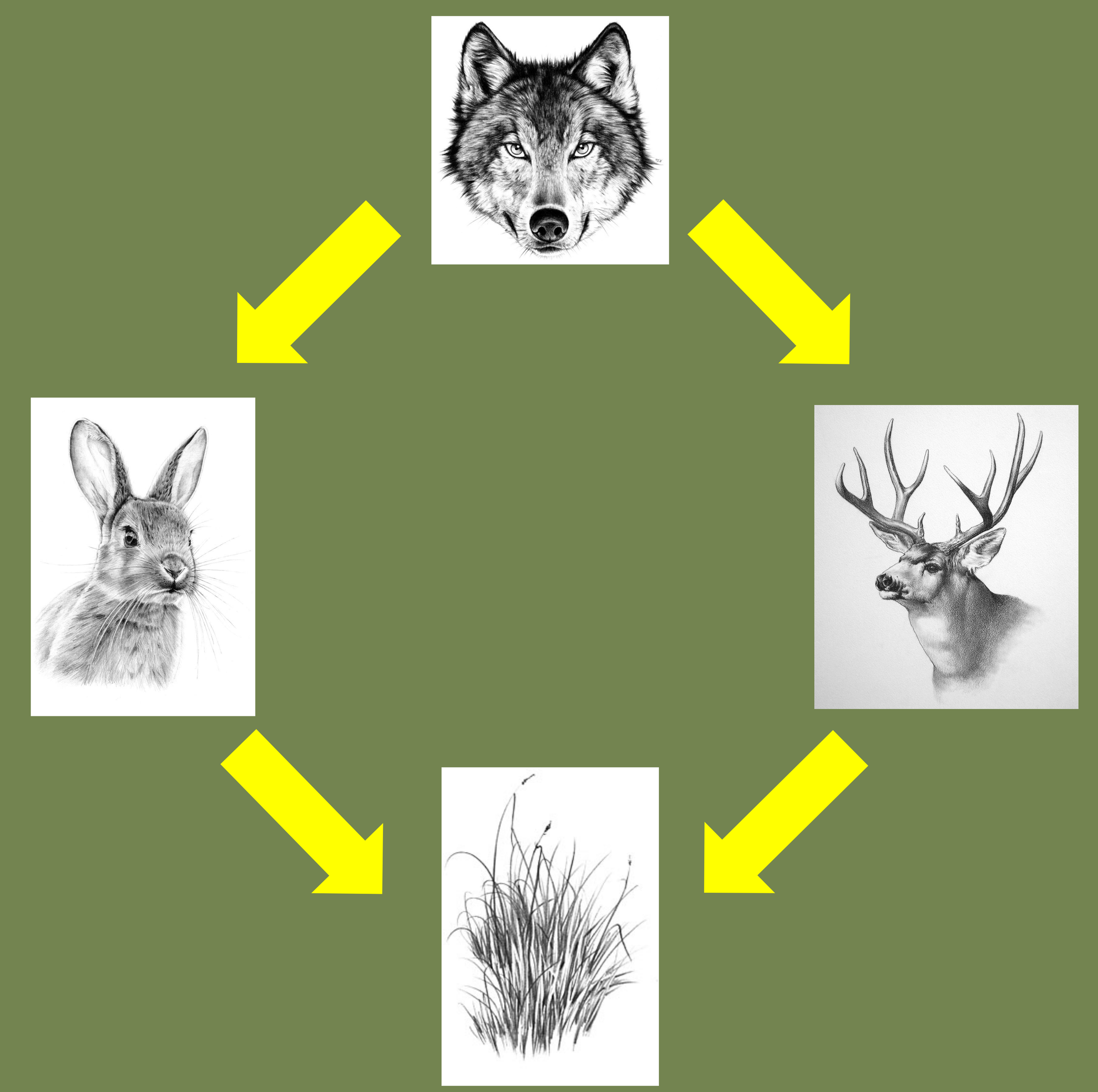
Some of the assumptions necessary in our model:

- Constant environmental factors (no time dependency). This includes:
  - Constant seasons and ability to reproduce
  - Equal availability of grass to elk and rabbits
  - Year over year there is no environmental factor that impacts species populations
  - Species are unable to leave, this is a closed environmental system
  - No other species interact with these 4 organisms
- Independence of species interaction (e.g. a wolf finding one elk does not impact its ability to find another or find a rabbit)
- Constant biomass transfusion rate (e.g. all elk have the same biomass transfusion to wolves)
- The animals are able to reproduce from age 0 and don't lose their reproductive abilities
- Animals within a species are uniform, there are no mutations that give evolutionary advantages and there is no distinction between male and female
- Initial populations are based on Yellowstone published statistics, and are therefore an approximation on what we expect could have been present in the park
- We assume the natural death rate to be a constant fraction of the organism population based on age expectancy

We find these assumptions to be reasonable and based on common knowledge as well as the literature. However, we recognize the shortcomings that result from such assumptions and will attempt to keep conclusions within reasonable scope.

NETWORK

VISUAL REPRESENATION



SYSTEM OF EQUATIONS

$$\frac{dP}{dt} = \xi_1 HP + \xi_2 RP + d_p P$$
$$\frac{dH}{dt} = \lambda HP + \delta HG + d_h H$$
$$\frac{dR}{dt} = \omega RP + \alpha RG + d_r R$$
$$\frac{dG}{dt} = \beta_1 HG + \beta_2 RG + \phi G$$

KEY

ξ_1: transfusion rate between elk and wolf
ξ_2: transfusion rate between rabbit and wolf
d_p, d_h, d_r: death rates for wolves, elk, and rabbits
λ: kill rate of elk for wolves
δ: transfusion rate between grass and elk
ω: kill rate of rabbits for wolves
α: transfusion rate between grass and rabbits
β_1: kill rate of grass for elk
β_2: kill rate of grass for rabbits
φ: reproduction rate for grass

STABILITY ANALYSIS

Before Wolves
(P=0)

$$\frac{dH}{dt} = +\delta G + d_h = 0$$
$$\frac{dR}{dt} = \alpha G + d_r = 0$$
$$\frac{dG}{dt} = \beta_1 H + \beta_2 R + \phi = 0$$
$$G = -\frac{d_h}{\delta}, G = -\frac{d_r}{\alpha}$$

Unless $d_h/\delta = d_r/\alpha$ or H or R=0 there is no system equilibrium.

After wolf reintroduction
($G, P, R, H \neq 0$)

$$\frac{dP}{dt} = \xi_1 H + \xi_2 R + d_p = 0$$
$$\frac{dH}{dt} = \lambda P + \delta G + d_h = 0$$
$$\frac{dR}{dt} = \omega P + \alpha G + d_r = 0$$
$$\frac{dG}{dt} = \beta_1 H + \beta_2 R + \phi = 0$$

$$\begin{bmatrix} \xi_1 & \xi_2 & 0 & 0 & -d_p \\ \beta_1 & \beta_2 & 0 & 0 & -\phi \\ 0 & 0 & \omega & \alpha & -d_r \\ 0 & 0 & \lambda & \delta & -d_h \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{d_p\beta_2 - \xi_2\phi}{\xi_2\beta_1 - \xi_1\beta_2} \\ 0 & 1 & 0 & 0 & \frac{-\phi\xi_1 + d_p\beta_1}{\xi_1\beta_2 - \xi_2\beta_1} \\ 0 & 0 & 1 & 0 & \frac{d_h * \alpha - \delta d_r}{\omega\delta - \lambda\alpha} \\ 0 & 0 & 0 & 1 & \frac{-d_r\lambda + d_h\omega}{\lambda\alpha - \omega\delta} \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{d_p\beta_2 - \xi_2\phi}{\gamma} \\ 0 & 1 & 0 & 0 & \frac{\phi\xi_1 - d_p\beta_1}{\gamma} \\ 0 & 0 & 1 & 0 & \frac{d_h * \alpha - \delta d_r}{\rho} \\ 0 & 0 & 0 & 1 & \frac{d_r\lambda - d_h\omega}{\rho} \end{bmatrix}$$

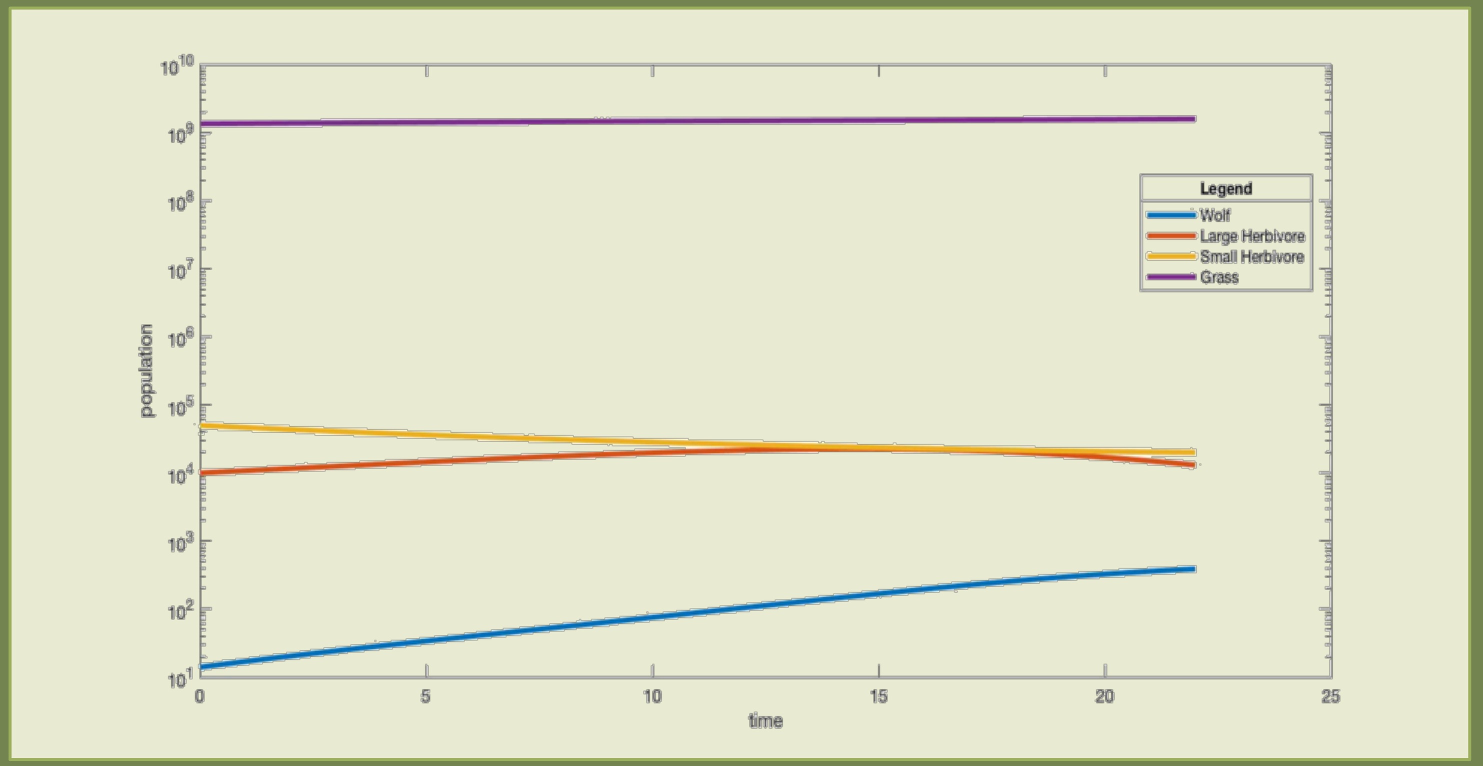
$$H = \frac{d_p\beta_2 - \xi_2\phi}{\gamma}$$
$$R = \frac{\phi\xi_1 - d_p\beta_1}{\gamma}$$
$$P = \frac{d_h * \alpha - \delta d_r}{\rho}$$
$$G = \frac{d_r\lambda - d_h\omega}{\rho}$$

After wolf reintroduction, system equilibrium may exist.

NUMERICAL RESULTS

RUNGE-KUTTA 4

Graphical Time Series Analysis



22 Years Later: Comparison with Initial Values

Animal	Start (1995)	Today (2017)	% change	Current Trajectory
Wolf	14	383	+2,636%	Increasing
Elk	10000	13, 900	+39%	Decreasing
Rabbit	50000	20000	-60%	Flat
Grass	1.35×10^9	1.59×10^9	+17.8%	Increasing

LIMITATIONS

- Model is highly chaotic; the parameter space can generate drastically different and counterintuitive outcomes for minute shifts in constants
- Model has a limited time horizon
- Our model only accounts for predator-prey relations between a small subset of creatures, doesn't acknowledge other forms of ecological relationships

CONCLUSIONS

- Introducing Predators into unstable ecological systems can stabilize growth rates
- Historical trends can be observed through our model
- Adding predators not only effects their prey but the entire ecosystem
- Chaotic systems are often unpredictable but, in this case the particular parameter space allows a near stable environment
- Even a small perturbation of an ecological system can have an effect multiple orders of magnitude greater than the initial perturbation