CHAOS 27, 101102 (2017)

Coherence resonance in a network of FitzHugh-Nagumo systems: Interplay of noise, time-delay, and topology

Maria Masoliver,[1] Nishant Malik,[2] Eckehard Schöll,[3] and Anna Zakharova[3]
[1]*Universitat Politècnica de Catalunya, Colom 11, Terrassa, ES-08222 Barcelona, Spain*
[2]*Department of Mathematics, Dartmouth College, Hanover, New Hampshire 03755, USA*
[3]*Institut für Theoretische Physik, Technische Universität Berlin, Hardenbergstr. 36, 10623 Berlin, Germany*

(Received 4 September 2017; accepted 22 September 2017; published online 10 October 2017)

We systematically investigate the phenomena of coherence resonance in time-delay coupled networks of FitzHugh-Nagumo elements in the excitable regime. Using numerical simulations, we examine the interplay of noise, time-delayed coupling, and network topology in the generation of coherence resonance. In the deterministic case, we show that the delay-induced dynamics is independent of the number of nearest neighbors and the system size. In the presence of noise, we demonstrate the possibility of controlling coherence resonance by varying the time-delay and the number of nearest neighbors. For a locally coupled ring, we show that the time-delay weakens coherence resonance. For nonlocal coupling with appropriate time-delays, both enhancement and weakening of coherence resonance are possible. *Published by AIP Publishing.*
https://doi.org/10.1063/1.5003237

The FitzHugh-Nagumo system is a paradigmatic model which describes the excitability and spiking behavior of neurons. It has various applications ranging from biological processes to nonlinear electronic circuits. In the excitable regime under the influence of noise, this model exhibits the counterintuitive phenomenon of coherence resonance. It means that there exists an optimum intermediate value of the noise intensity for which noise-induced oscillations become most regular. We investigate coherence resonance in a network of delay-coupled FitzHugh-Nagumo elements with local, nonlocal, and global coupling topologies. Networks with nonlocal topology are inspired by neuroscience, as they emulate the observation that strong interconnections between neurons are typical within a certain range while fewer connections exist at longer distances. We illustrate that the interaction between the network topology, the time-delay in the coupling, and the noise leads to a rich oscillatory dynamics. In particular, we demonstrate that the regularity of this dynamics is controllable, i.e., one can enhance or weaken coherence resonance by varying the coupling and delay time.

I. INTRODUCTION

All natural processes are inevitably affected by internal and external random fluctuations, i.e., noise. Even a relatively low noise intensity can significantly influence the behavior of a dynamical system. In nonlinear systems, noise can play a constructive role and give rise to new dynamic behavior, e.g., stochastic bifurcations, stochastic synchronization, or coherence resonance.[1–4] The counterintuitive effect of coherence resonance describes the non-monotonic behavior of the regularity of noise-induced oscillations in the excitable regime. This results in an optimum response in terms of the regularity of the oscillations for an intermediate noise strength.

In addition to noise, the presence of time-delay can essentially change the dynamics of a real-world system. Time-delay naturally arises in many processes, including population dynamics, chemical reactions, and lasers.[5] Interestingly, time-delay has been used not only to describe these processes but also to control them. For instance, when introduced in a nonlinear dynamical system, it can control deterministic chaos.[6] Delay can control noise-induced oscillations as well and consequently effects such as stochastic resonance and coherence resonance. A passive self-adaptive method for controlling noise-induced oscillations already exists, and delayed feedback previously used to control deterministic chaos forms the basis of this approach.[7–11] Since then, several studies have been conducted on both excitable and non-excitable systems, as well as on single and coupled oscillators.[12–16] These studies illustrate that delayed feedback effectively manipulates the properties of coherence resonance and adjusts the timescales of oscillations. Previous studies have revealed that introduction of time-delayed feedback in a single system can control coherence resonance.[17,18] In many systems, there are physical reasons for including time-delay in their modeling. For example, in neuroscience combining coupling with time-delayed feedback is a convenient approach to describe signal transmission in neuronal networks, i.e., the propagation delay of action potentials between neurons. Meanwhile, modeling studies have shown that the presence of time-delay coupling can regulate the dynamics in networks, including stochastic synchronization in noise-affected systems and coupled lasers.[19–23]

The objective of this work is to investigate the interplay between noise, delay, and network topology of time-delay coupled neurons, where the FitzHugh-Nagumo model in the excitable regime represents the local dynamics of each neuron. In particular, we are interested in the phenomena of coherence resonance.[3,8–10,12,13,17,24–33] Thus far, the control of coherence resonance has been studied in single FitzHugh-

Nagumo and in two coupled FitzHugh-Nagumo oscillators with time-delayed feedback.[12] In contrast, here we aim to investigate the control of coherence resonance in a network of delay-coupled FitzHugh-Nagumo oscillators.

The organization of this paper is as follows: In Sec. II, we introduce the model and describe the behavior of a single FitzHugh-Nagumo oscillator. In Sec. III, we characterize the regimes of delay-induced oscillations in the deterministic case. Next, in Sec. IV, we discuss the stochastic case but without the time-delay. We introduce different measures of coherence resonance and present an analysis of coherence resonance in a network of oscillators without the delayed coupling. We also explore the dependence of coherence resonance on the coupling parameters and on the bifurcation parameter. Finally, in Sec. V, we investigate the interplay of noise, delayed coupling, and network topology. We explore in detail how the time-delay and nearest neighbor coupling influence the coherence resonance, including the cases when underlying networks have complex topologies. Finally, we conclude in Sec. VI with a summary of the results.

II. MODEL

Throughout this paper, the model considered is a network of N coupled FitzHugh-Nagumo oscillators. A FitzHugh-Nagumo oscillator is a minimalistic prototypical model of an excitable system.[34,35] Excitable systems possess a single stable rest state and remain in the rest state unless perturbed by a sufficiently strong external input. Once perturbed, the system leaves the rest state and passes through the firing and the refractory states. The external driving has only a weak influence on the firing and refractory state.[26] Nonlinear dynamical systems exhibiting the above properties have been proposed as models for neuronal spike generation. In neuroscience, the large excursion of the system's variables due to strong external perturbation (forcing the system to leave the rest state) is called a spike and their occurrence as firing. The excitability of a neuron can be classified into two categories, namely, type I and type II. Whereas type-I neurons undergo a saddle-node infinite period bifurcation, type-II neurons undergo a supercritical Hopf bifurcation.[26,36–38] A phenomenological description of this distinction also exists in the classical work of Hodgkin and Huxley.[39] The FitzHugh-Nagumo oscillator has been employed to model type-II neurons.

The following set of equations describes a ring network of N FitzHugh-Nagumo oscillators:

$$\begin{aligned}\epsilon\dot{u}_i &= u_i - \frac{u_i^3}{3} - v_i + \frac{\sigma}{2P}\sum_{j=i-P}^{i+P}\left[u_j(t-\tau) - u_i(t)\right],\\ \dot{v}_i &= u_i + a + \sqrt{2D}\xi_i(t), \quad i = 1,\dots,N, \end{aligned} \tag{1}$$

where u_i and v_i are dimensionless variables. The voltage-like variable u_i allows for regenerative self-excitation through positive feedback, i.e., it is an activator variable; v_i is a recovery-like variable and provides a slower negative feedback, i.e., it is an inhibitor variable. The index i stands for the node i in the ring network of N oscillators. The time-scale parameter ϵ is usually much smaller than 1 for neuronal models; here, we set $\epsilon = 0.01$. P denotes the number of nearest neighbors to each side. For a ring, every node has the same number of connections; this gives rise to two limiting cases of local and global coupling, $P = 1$ and $P = (N-1)/2$ (for odd N), respectively. Note that for sufficiently large N, global coupling can be approximated by $P = N/2$. When $1 < P < N/2$, we call it non-local coupling. Thus, P acts as a control parameter for the topology of the underlying network. σ is the constant coupling strength, and the coupling term has the form of classical diffusive coupling, i.e., the coupling vanishes if the variables u_i and u_j are identical. τ is the propagation delay. D stands for the noise intensity. In this work, we use Gaussian white noise represented by $\xi(t)$ with $\langle\xi(t)\rangle = 0$ and $\langle\xi(t)\xi(t')\rangle = \delta(t-t')$ for $t \neq t'$.[40] a is the deterministic bifurcation parameter. A single FitzHugh-Nagumo system in the deterministic case ($D = 0$) undergoes a supercritical Hopf bifurcation at $a = 1$. For $|a| < 1$, the system is in the oscillatory regime where the steady state is unstable and self-sustained oscillations are observed. For $|a| > 1$, the system is in the excitable regime and characterized by a locally stable steady state.

III. DETERMINISTIC CASE: IMPACT OF TIME-DELAY

To study the effect of delayed coupling on coherence resonance, the system must be in the parameter regime where no delay-induced oscillations exist. However, for certain time-delays and coupling strengths, delayed coupling induces self-sustained oscillations between two coupled FitzHugh-Nagumo oscillators, even when both oscillators are in the excitable regime (stable steady state) and there is no external noise applied.[19,41] A saddle-node bifurcation resulting in a pair of stable and unstable limit cycles generates these oscillations.[19] To identify parameter regimes where delay coupling induced oscillations are absent, we first study a ring network of N FitzHugh-Nagumo oscillators in the deterministic regime by setting $D = 0$ in Eq. (1). We numerically integrate Eq. (1) for different values of τ and σ and calculate the interspike interval or the oscillation period T of synchronized oscillations. The results for $a = 1.05$ and $a = 1.3$ are shown in Figs. 1(a) and 1(b), respectively, where T is plotted in the parameter space of coupling strength σ and delay time τ. Figure 1(a) with $a = 1.05$ is closer to the Hopf bifurcation point, and we observe that further away from the bifurcation point [Fig. 1(b) with $a = 1.3$], we require larger delay τ and coupling strength σ to obtain delay-induced oscillations. The black region in Figs. 1(a) and 1(b) stands for the absence of delay-induced oscillations: this is the regime on which we focus in this work.

For both Figs. 1(a) and 1(b), we use $P = 4$; however, it can be shown that the region of delay-induced oscillations is independent of the ring topology and system size: Consider the delayed-coupling term in Eq. (1): $\frac{\sigma}{2P}\sum_{j=i-P}^{i+P}[u_j(t-\tau) - u_i(t)], j \neq i$. Since delay-induced oscillations are synchronized, i.e., $u_1(t-\tau) = \cdots = u_N(t-\tau) \equiv u_{sync}(t-\tau)$ and $u_1(t) = \cdots = u_N(t) \equiv u_{sync}(t)$, where u_{sync} is the synchronized solution, we can simplify the delayed coupling term as

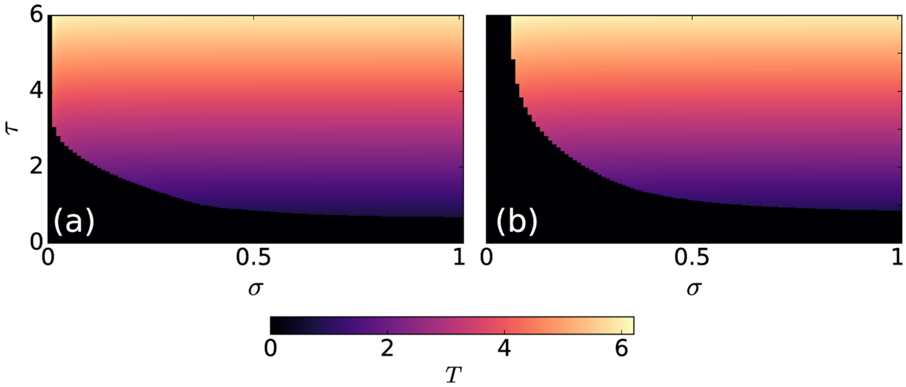

FIG. 1. Regime of delay-induced oscillations in the (τ, σ) plane for different values of the bifurcation parameter: (a) $a = 1.05$ and (b) $a = 1.3$. The period of oscillations T is color coded and corresponds to $T = \tau + \delta$ with small $\delta > 0$. The black region denotes the absence of delay-induced oscillations. The initial history function corresponds to a spike for all oscillators. Other parameters: $\epsilon = 0.01$, $N = 100$, $P = 4$, and $D = 0$.

$\sigma[u_{sync}(t-\tau) - u_{sync}(t)]$. Rewriting Eq. (1) for the deterministic case, we find

$$\epsilon \dot{u}_{sync} = u_{sync} - \frac{u_{sync}^3}{3} - v_{sync} + \sigma\left[u_{sync}(t-\tau) - u_{sync}(t)\right],$$
$$\dot{v}_{sync} = u_{sync} + a. \tag{2}$$

It is observed that this equation is independent of both the nearest neighbor number P and the number of oscillators N. Thus, the regime of delay-induced synchronized oscillations is independent of the ring topology and system size.

IV. COHERENCE RESONANCE

Pikovsky and Kurths[24] coined the term *coherence resonance* to characterize the emergence of relatively coherent oscillations in a FitzHugh-Nagumo system at an optimal noise intensity. Since then, this phenomenon has been extensively studied in various nonlinear models. Several different measures exist in the literature for quantifying coherence resonance, such as the correlation time, the signal-to-noise-ratio, and the normalized standard deviation of the interspike interval.[3,13,24] In this work, we will use the last one. It is defined as $R = \frac{\sqrt{\langle t_{ISI}^2 \rangle - \langle t_{ISI} \rangle^2}}{\langle t_{ISI} \rangle}$, where t_{ISI} is the time between two subsequent spikes and $\langle \cdots \rangle$ indicates the average over the time series. A system undergoing coherence resonance will show a pronounced minimum in the value of R.[24] The above definition of R is limited to characterizing coherence resonance for a single FitzHugh-Nagumo oscillator. For a network of oscillators, coherence resonance can be measured by redefining R as follows: $R = \frac{\sqrt{\overline{\langle t_{ISI}^2 \rangle} - \overline{\langle t_{ISI} \rangle}^2}}{\overline{\langle t_{ISI} \rangle}}$, where the overline indicates the additional average over nodes. We will refer to $\overline{\langle t_{ISI} \rangle}$ as the period of the system T. Moreover, we refer to the period that the system shows under coherence resonance as the intrinsic period of the system and denote it by T_o.

Next, we study the role of noise intensity D and coupling strength σ in inducing coherence resonance in a network of locally coupled ($P = 1$) FitzHugh-Nagumo oscillators without delay. We measure R in two different parameter settings, first we increase D, keeping all parameters fixed and second we increase σ, keeping all the other parameters fixed. The results are shown in Figs. 2(a) and 2(b); note that the x-axis is logarithmic. In Fig. 2(a), both curves for $a = 1.05$ and $a = 1.3$ have a minimum, i.e., both cases show coherence resonance at two different noise intensities D. It is worth noting here that if the system is closer to the Hopf bifurcation point, i.e., for $a = 1.05$, it requires lower noise intensity for coherence resonance to occur. On the other hand, if the system is further away from the Hopf bifurcation point, i.e., for $a = 1.3$, the system requires higher noise intensity. We observe $D = 0.001$ for the former and $D = 0.079$ for the latter case. In Fig. 2(a), we have set $\sigma = 0.1$, $P = 1$, and $N = 100$. To study the effects of coupling strength on the above observed coherence resonance, we measure R as σ is varied in two different parameter settings: first, for $a = 1.05$ and $D = 0.001$ and

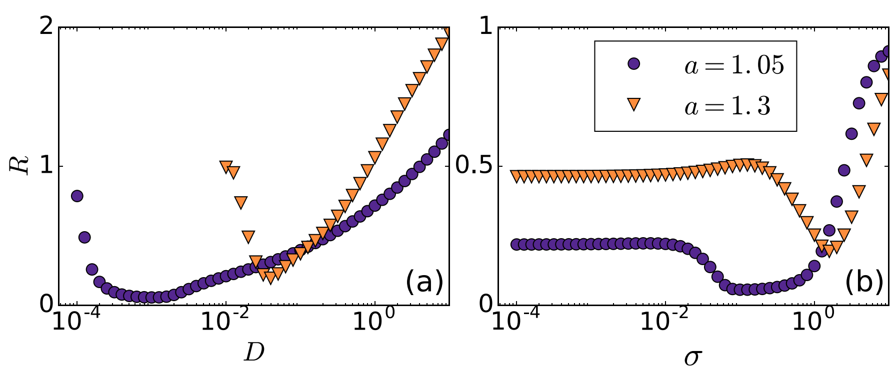

FIG. 2. Normalized standard deviation of the interspike interval R for two different values of the bifurcation parameter $a = 1.05$ (circles) and $a = 1.3$ (triangles): (a) for fixed coupling strength $\sigma = 0.1$ and varying noise intensity D and (b) for fixed noise intensities $D = 0.001$ (for $a = 1.05$) and $D = 0.079$ (for $a = 1.3$) and varying coupling strength σ. The results are obtained by integrating Eq. (1) over 10 000 time units and then averaging over time, oscillators, and realizations (for 20 simulations each). Note the logarithmic scale for the x-axis. Other parameters: $\epsilon = 0.01$, $P = 1$, $N = 100$, and $\tau = 0$.

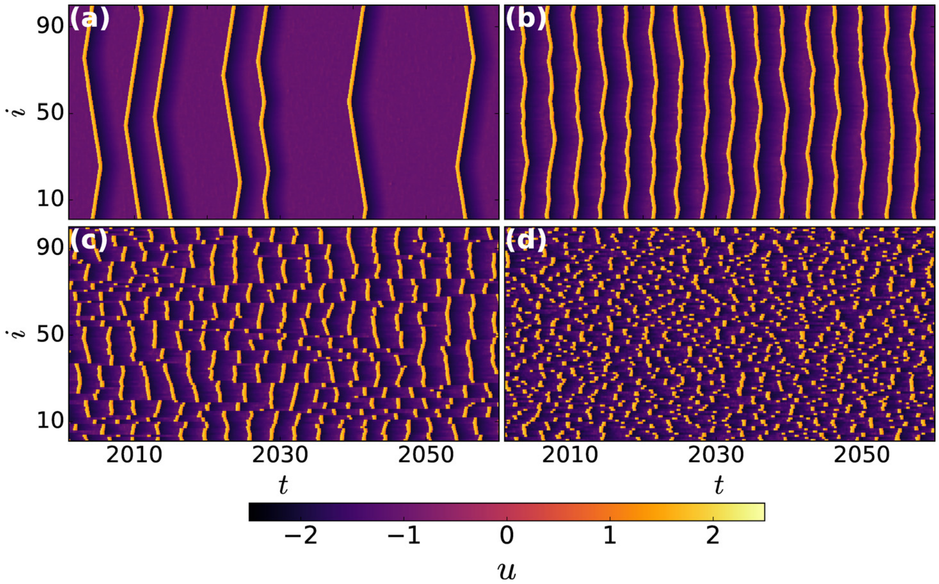

FIG. 3. Space-time plots for $a = 1.05$ at different noise intensities: (a) $D = 0.00012$, (b) $D = 0.001$, (c) $D = 0.005$, and (d) $D = 0.05$. Other parameters: $\epsilon = 0.01$, $\tau = 0$, $P = 1$, $N = 100$, and $\sigma = 0.1$.

second, for $a = 1.3$ and $D = 0.079$. The results are plotted in Fig. 2(b). We observe for the case $a = 1.05$ and $D = 0.001$ that coherence resonance is enhanced when $0.1 \leq \sigma < 1$. In Fig. 2(b), we have set $P = 1$ and $N = 100$. Several other works have also shown that coherence resonance can be enhanced by choosing appropriate coupling strengths. For example in Refs. 25 and 42, it was shown that some choices of coupling strength increase coherence resonance in an array of non-identical FitzHugh-Nagumo oscillators; in Ref. 31, a similar feature was observed in the case of weighted scale-free networks.

To visualize the above observations, in Figs. 3(a)–3(d), we depict space-time plots for different noise intensities. For small noise $D = 0.00012$, in Fig. 3(a), we observe that the system is spiking irregularly, but still it is in a highly synchronized state. This implies that the deterministic coupling dominates the system dynamics for small noise intensities. As the noise intensity is increased to an optimal value $D = 0.001$, in Fig. 3(b), we observe highly regular synchronous spiking, and this is the parameter regime where we observe coherence resonance. Once noise exceeds its optimal value, and in Fig. 3(c), the system exhibits cluster synchronization, i.e., several clusters of synchronously spiking oscillators are formed. When external noise is further increased, in Fig. 3(d), cluster synchronization disappears and each node oscillates individually, driven by its own noise.

So far, we have only discussed coherence resonance in a ring network with $P = 1$ (every node has exactly two neighbors). To explore the impact of P, we fix $\sigma = 0.1$ and $a = 1.05$ and study the system with four different values of P, namely, $P = 4$, $P = 12$, $P = 25$, and $P = 50$ (all to all connected network). For each case, we calculate the noise intensity D_o for which we observe coherence resonance. Also, we evaluate the corresponding values of R and T and denote them as R_o and T_o, respectively. We summarize our findings in Table. I. We notice that as P is increased, D_o and R_o decrease, indicating that we require lower noise intensity to observe stronger coherence resonance for higher P. In contrast to a single FitzHugh-Nagumo system, coherence-resonance in a network occurs at lower values of the noise intensity.

V. INTERPLAY OF TOPOLOGY AND DELAYED COUPLING

In this section, we explore the effects of topology and delayed coupling on noise-induced oscillations. In a single FitzHugh-Nagumo oscillator, a time-delay can either enhance or suppress coherence resonance.[9] If the delay is $\tau = nT_o$, then for an integer n, coherence resonance increases, whereas for half-integer n, it is weakened.[9] In contrast to Refs. 8 and 9, where a single FitzHugh-Nagumo oscillator is analyzed, we study a network of N delay-coupled FitzHugh-Nagumo oscillators. Therefore, we will illustrate not only the influence of time-delay but also the topology.

We will investigate the effects of $\tau = \frac{1}{2}T_o$ and $\frac{1}{3}T_o$ on a network of oscillators described by $P = 1$, $P = 4$, $P = 25$, and $P = 50$. It is important to note that T_o refers to the period of the network with particular topology when it exhibits coherence resonance. Hence, for each P, we have a different T_o. Next, we present the results on the influence of coupling topology on coherence resonance. We divide these results into three sections according to the coupling topology discussed. Section V A examines the locally-coupled ring, Sec. V B discusses the non-locally and globally coupled ring, and in Sec. V C, we consider networks with complex topologies.

TABLE I. Values of parameters D, T, and R at coherence resonance when P (number of nearest neighbors) is varied. Other parameters: $\epsilon = 0.01$, $a = 1.05$, $\sigma = 0.1$, and $N = 100$.

	$P = 1$	$P = 4$	$P = 12$	$P = 25$	$P = 50$
D_o	0.001	0.001	0.0008	0.0008	0.0008
T_o	3.53	3.51	3.53	3.61	3.62
R_o	0.06	0.04	0.032	0.029	0.029

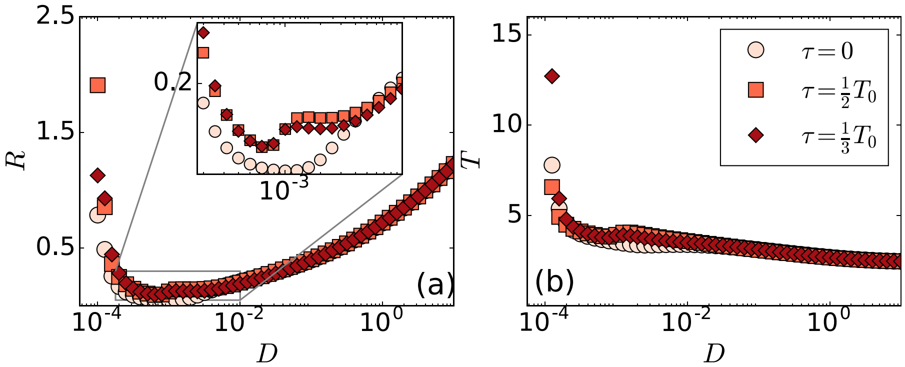

FIG. 4. (a) R vs. noise intensity D and (b) period of oscillations T vs. noise intensity D for different time-delays in the case of a locally coupled ring, i.e., $P=1$. The inset in panel (a) shows a zoom-in view of the indicated region. Other parameters: $\epsilon = 0.01$, $a = 1.05$, $N = 100$, and $\sigma = 0.1$.

A. Locally coupled ring

First, we study $P=1$, i.e., a locally coupled ring. We plot R and the period of oscillations T vs. noise intensity D in Figs. 4(a) and 4(b), respectively. The corresponding values of R_o, D_o, and T_o are listed in the first column of Table II. Comparing the values of R_o for $P=1$ without time-delay (Table I) and in the presence of time-delay (Table II), one can see that for $\tau = \frac{1}{2}T_o$ and $\frac{1}{3}T_o$, coherence resonance is slightly weakened. Also, coherence resonance occurs at the same noise intensity D_o, with almost the same minimal value R_o for both delays. Once the noise is sufficiently large, it overtakes the dynamics of the network, and delayed-coupling does not play a role any longer. As shown in Fig. 4(a), for both τ values, the system undergoes a local minimum in R, and a further increase in noise intensity leads to a monotonic increase in R.

B. Non-locally and globally coupled ring

Now, we study a non-locally coupled ($1 < P < 50$) and a globally coupled ($P=50$) ring network. The non-locally and globally coupled ring leads to two different outcomes: while coherence resonance is enhanced for $\tau = \frac{1}{2}T_o$, and it is weakened for $\tau = \frac{1}{3}T_o$, compared with the undelayed case (see the values of R_o in Tables I and II). The corresponding numerical results are displayed in columns two to four in Table II and plotted in Figs. 5(a)–5(f).

For $\tau = \frac{1}{2}T_o$, coherence resonance is strengthened as P increases, with the strongest coherence resonance being observed for global coupling [see Table II or Figs. 5(a) and 5(c)]. In contrast, for $\tau = \frac{1}{3}T_o$, coherence resonance is weakened with increasing P [see Table II or Fig. 5(e)]. Also, when $\tau = \frac{1}{2}T_o$, in Table II, we observe that coherence resonance occurs at smaller noise intensities compared to $P=1$. It should be noted that for both $\tau = \frac{1}{2}T_o$ and $\tau = \frac{1}{3}T_o$, the values of T_o are higher than one observed for $\tau = 0$ in Table I.

In Table II, for $\tau = \frac{1}{3}T_o$, we note that increasing P leads to increasing R_0, i.e., the irregularity of the motion increases. This particular observation suggests that for $\tau = \frac{1}{3}T_o$, delayed-coupling induces irregularity in the oscillations. It destabilizes the time interval between successive spikes and nodes. Hence, it increases the range of the variation of the period T under the change of noise strength [also see Figs. 5(b), 5(d), and 5(f)].

TABLE II. Values of parameters D, T, and R at coherence resonance when P (number of nearest neighbors) and τ (time-delay) are varied. Other parameters: $\epsilon = 0.01$, $a = 1.05$, $\sigma = 0.1$, and $N = 100$.

	$P=1$	$P=4$	$P=25$	$P=50$
	Delay-coupled network $\tau = \frac{1}{2}T_o$			
D_o	0.0006	0.0004	0.00025	0.0002
T_o	3.85	3.66	3.75	3.8
R_o	0.094	0.036	0.01	0.007
	Delay-coupled network $\tau = \frac{1}{3}T_o$			
D_o	0.0006	0.0004	0.0005	0.0006
T_o	3.79	3.96	4.18	4.26
R_o	0.096	0.092	0.127	0.159

C. Networks with complex topologies

Here, we discuss the examples of networks with complex topologies, namely, the Erdős-Rényi random network and the small-world network. We employed the Watts-Strogatz model to generate a small-world network.[43]

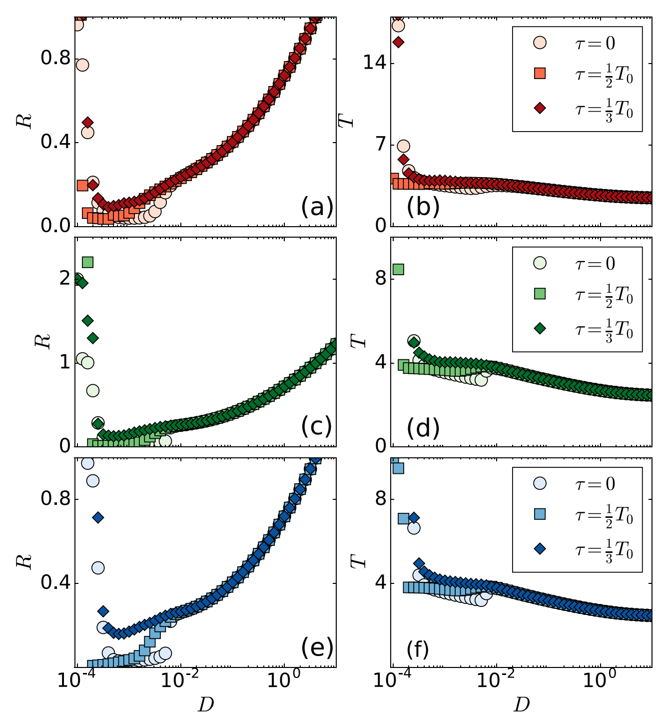

FIG. 5. Same as Fig. 4 for a non-locally coupled ring: (a) and (b) $P=4$; (c) and (d) $P=25$; and (e) and (f) globally coupled ring ($P=50$). (a), (c), and (e): R vs. D and (b), (d), and (f): oscillation period T vs. D for different time-delays (see the legend for specific values). Other parameters are as in Fig. 4.

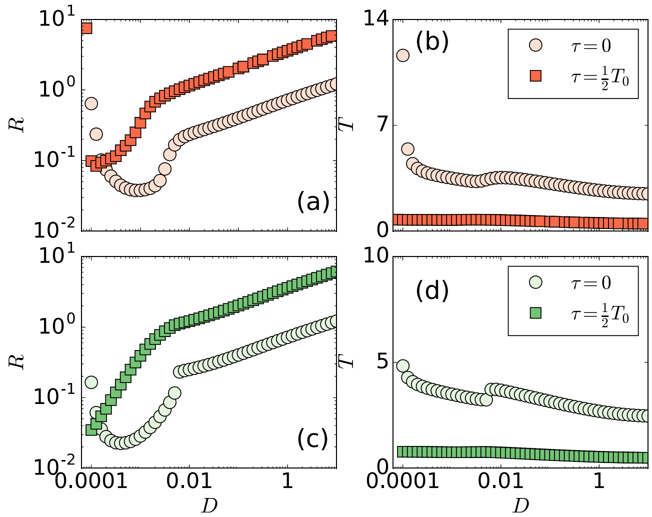

FIG. 6. (a) and (c) R vs D; (b) and (d) T vs D for small-world (a) and (b) and Erdős-Rényi random (c) and (d) networks for different delay times: $\tau=0$ and $\tau=\frac{1}{2}T_{opt}$. Other parameters: $a=1.05$, $N=1000$, and $\sigma=0.1$.

For $\tau=0$, coherence resonance occurs for both cases [Figs. 6(a) and 6(c)]. However, in the presence of time delay for the small-world network, coherence resonance is weakened [Fig. 6(a) and 6(b)], whereas for the Erdős-Rényi random network, it is suppressed [Figs. 6(c) and 6(d)]: R only grows while D increases.

VI. SUMMARY AND CONCLUSIONS

We have systematically studied coherence resonance in a network of delay-coupled FitzHugh-Nagumo oscillators and have presented a detailed analysis of rich dynamics emerging due to the interactions between noise, time-delayed coupling, and topology. First, we demonstrated that in a ring network of deterministic FitzHugh Nagumo delay-coupled oscillators, the regions of delay-induced oscillations are independent of the number of nearest neighbors P and system size. Furthermore, we showed that these regions depend on the bifurcation parameter a and they grow when a is further away from the Hopf bifurcation.

Next, we considered the stochastic case without time-delay, i.e., $D\neq 0$ and $\tau=0$. We observed that coherence resonance can be enhanced or weakened by the coupling strength σ. With increasing σ, we have found a minimum in the normalized variance of the interspike interval R, i.e., coherence resonance is strengthened. We studied coherence resonance for two different values of the bifurcation parameter, viz., $a=1.05$ and $a=1.3$ (with $P=1$), and found that larger noise intensity is required to observe coherence resonance in the latter case. Moreover, for $a=1.3$, higher coupling strength is needed for coherence resonance. That is why changing the system from $a=1.05$ to $a=1.3$ increases the range of no-delay-induced oscillations; coherence resonance requires higher noise and coupling. Therefore, keeping the system at $a=1.05$ is better suited to study the system, as it is more sensitive to noise, the number of nearest neighbor P, and coupling strength.

On including non-zero time-delay into the system, several new features emerged. We explored the system with two different time-delays, namely, $\tau=\frac{1}{2}T_o$ and $\tau=\frac{1}{3}T_o$. Whereas for a locally coupled ring ($P=1$), delay-coupling weakens coherence resonance for both values of τ, in the case of a non-locally ($1<P<50$) and globally ($P=50$) coupled ring, we found different results depending on τ. For $\tau=\frac{1}{2}T_o$, an enhancement of coherence resonance is observed, while for $\tau=\frac{1}{3}T_o$, coherence resonance is weakened. This is due to the influence of an *indirect* coupling: node i is directly coupled to $2P$ nodes, and additionally, it is *indirectly* coupled to $(2P)^2$ neighbors with a delayed coupling of 2τ. Hence, for $\tau=\frac{1}{2}T_o$, the total propagation delay is equivalent to T_o and for $\tau=\frac{1}{3}T_o$ to $\frac{2}{3}T_o$, leading to the enhancement or weakening of coherent dynamics.

ACKNOWLEDGMENTS

This work was supported by Deutsche Forschungsgemeinschaft in the framework of Collaborative Research Center SFB 910.

[1]C. S. Zhou and J. Kurths, Phys. Rev. Lett. **88**, 230602 (2002).
[2]N. Malik, B. Ashok, and J. Balakrishnan, Eur. Phys. J. B **74**, 177 (2010).
[3]G. Hu, T. Ditzinger, C. Z. Ning, and H. Haken, Phys. Rev. Lett. **71**, 807 (1993).
[4]Y. Wang, D. T. W. Chik, and Z. D. Wang, Phys. Rev. E **61**, 740 (2000).
[5]T. Erneux, *Applied Delay Differential Equations* (Springer, 2009).
[6]*Handbook of Chaos Control*, 2nd ed., edited by E. Schöll and H. G. Schuster (Wiley-VCH, Weinheim, 2008).
[7]C. Masoller, Phys. Rev. Lett. **88**, 034102 (2002).
[8]N. B. Janson, A. G. Balanov, and E. Schöll, Phys. Rev. Lett. **93**, 010601 (2004).
[9]A. G. Balanov, N. B. Janson, and E. Schöll, Physica D **199**, 1 (2004).
[10]E. Schöll, A. G. Balanov, N. B. Janson, and A. B. Neiman, Stoch. Dyn. **5**, 281 (2005).
[11]C. Masoller, M. C. Torrent, and J. García-Ojalvo, Phys. Rev. E **78**, 041907 (2008).
[12]B. Hauschildt, N. B. Janson, A. G. Balanov, and E. Schöll, Phys. Rev. E **74**, 051906 (2006).
[13]P. M. Geffert, A. Zakharova, A. Vüllings, W. Just, and E. Schöll, Eur. Phys. J. B **87**, 291 (2014).
[14]T. Prager, H. P. Lerch, L. Schimansky-Geier, and E. Schöll, J. Phys. A **40**, 11045 (2007).
[15]W. Just, A. Pelster, M. Schanz, and E. Schöll, Philos. Trans. R. Soc. A **368**, 301–513 (2010).
[16]V. Flunkert, I. Fischer, and E. Schöll, Philos. Trans. R. Soc. A **371**, 20120465 (2013).
[17]P. M. Geffert, "Stochastic non-excitable systems with time-delay," *BestMasters 2015* (Springer-Spektrum, Wiesbaden, 2015).
[18]W. Just, P. M. Geffert, A. Zakharova, and E. Schöll, in *Control of Self-Organizing Nonlinear Systems*, edited by E. Schöll, S. H. L. Klapp, and P. Hövel (Springer, Berlin/Heidelberg, 2016)
[19]E. Schöll, G. Hiller, P. Hövel, and M. A. Dahlem, Philos. Trans. R. Soc. A **367**, 1079 (2009).
[20]P. Hövel, *Springer Theses* (Springer, Berlin, 2010)
[21]V. Flunkert, *Springer Theses* (Springer, Berlin, 2011)
[22]M. C. Soriano, J. García-Ojalvo, C. R. Mirasso, and I. Fischer, Rev. Mod. Phys. **85**, 421 (2013).
[23]A. Saha and U. Feudel, Phys. Rev. E **95**, 062219 (2017).
[24]A. Pikovsky and J. Kurths, Phys. Rev. Lett. **78**, 775 (1997).
[25]B. Hu and C. Zhou, Phys. Rev. E **61**, R1001(R) (2000).
[26]B. Lindner, J. García-Ojalvo, A. B. Neiman, and L. Schimansky-Geier, Phys. Rep. **392**, 321 (2004).
[27]R. Aust, P. Hövel, J. Hizanidis, and E. Schöll, Eur. Phys. J.: Spec. Top. **187**, 77 (2010).
[28]O. A. Rosso and C. Masoller, Phys. Rev. E **79**, 040106(R) (2009).
[29]A. Zakharova, T. Vadivasova, V. Anishchenko, A. Koseska, and J. Kurths, Phys. Rev. E **81**, 011106 (2010).
[30]A. Zakharova, A. Feoktistov, T. Vadivasova, and E. Schöll, Eur. Phys. J. Spec. Top. **222**, 2481 (2013).

[31]P. Balenzuela, P. Rué, S. Boccaletti, and J. García-Ojalvo, New. J. Phys. **16**, 013036 (2014).
[32]V. Semenov, A. Feoktistov, T. Vadivasova, E. Schöll, and A. Zakharova, Chaos **25**, 033111 (2015).
[33]N. Semenova, A. Zakharova, V. S. Anishchenko, and E. Schöll, Phys. Rev. Lett. **117**, 014102 (2016).
[34]R. FitzHugh, Biophys. J. **1**, 445 (1961).
[35]J. Nagumo, S. Arimoto, and S. Yoshizawa, Proc. IRE **50**, 2061 (1962).
[36]E. M. Izhikevich, Inter. J. Bifurcation and Chaos **10**(06), 1171–1266 (2000).
[37]J. Rinzel and G. B. Ermentrout, in *Methods in Neuronal Modeling*, edited by C. Koch and I. Segev (MIT Press, Cambridge, MA, 1989), pp. 251–291.
[38]J. Lehnert, *Springer Theses* (Springer, Berlin, 2016)
[39]A. L. Hodgkin and A. F. Huxley, J. Physiology **117**(4), 500–544 (1952).
[40]C. W. Gardiner, *Handbook of Stochastic Methods for Physics, Chemistry and the Natural Sciences* (Springer, Berlin, 2002)
[41]M. A. Dahlem, G. Hiller, A. Panchuk, and E. Schöll, Int. J. Bifurcation Chaos **19**, 745 (2009).
[42]O. Kwon and H. T. Moon, Phys. Lett. A **298**, 319 (2002).
[43]D. J. Watts and S. H. Strogatz, Nature **393**, 440–442 (1998).