Dynamically Modeling Predator-Prey Chases

Sudhanshu Chanpuriya

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Abstract

We survey several predator strategies for pursuit of fleeing prey. The physical framework is a two-dimensional plane on which the predator and prey are point masses which are each able to exert a propulsion force of fixed magnitude and also experience a friction force. The model strategy for the prey is attempting self-propulsion a fixed angle away from the direction of predator. The chase trajectories resulting from various predator strategies for picking self-propulsion direction are calculated in a variety of conditions.

Introduction

Model problem

We employ Newton's Second Law in two dimensions to track the positions of the animals over time. The drag force is assumed to be in the opposite direction from velocity with a magnitude proportional to velocity. Letting the mass, two-dimensional position, self-propulsion force magnitude (corresponding to strength), self-propulsion direction, and drag coefficient of the predator be denoted by M, $\mathbf{X}$, $\hat{\mathbf{R}}$, F, and D, and letting the same elements for the prey be denoted by m, $\mathbf{x}$, $\hat{\mathbf{r}}$, f, and d, we have

$$M\ddot{\mathbf{X}} = F\hat{\mathbf{R}} - D\dot{\mathbf{X}} \qquad m\ddot{\mathbf{x}} = f\hat{\mathbf{r}} - d\dot{\mathbf{x}}$$

where dot denotes derivatives with respect to time t, while $\hat{\mathbf{R}}$ and $\hat{\mathbf{r}}$ are unit vectors corresponding to the strategies of the predator and prey, respectively; these unit vectors are functions of the animal's own mass, strength, drag coefficient, position, and velocity, as well as the other animal's position and velocity. Finally, let L_c denote the the distance within which the predator is considered to have caught the prey.

Nondimensionalization

Note that, given the above equations, the maximum speed attainable by the predator, S_m, and the minimum time it would take the predator to reach this speed in the absence of drag, T_m, are given by

$$S_m = \frac{F}{D} \qquad T_m = \frac{M}{D}$$

with corresponding equations holding for the prey's s_m and t_m.

We nondimensionalize from the perspective of the predator, letting L_c and T_m be the characteristic length and time and defining nondimensionalized predator position, prey position, and time as

$$\mathbf{Y} = \frac{\mathbf{X}}{L_c} \qquad \mathbf{y} = \frac{\mathbf{x}}{L_c} \qquad \tau = \frac{t}{T_m}$$

This yields

$$\mathbf{Y}'' = A\hat{\mathbf{R}} - \mathbf{Y}' \qquad \mathbf{y}'' = b\left(a\hat{\mathbf{r}} - \mathbf{y}'\right)$$

where prime denotes derivatives with respect to τ, and the nondimensional parameters A, a, and b are given by

$$A = \frac{MF}{D^2 L_c} = \frac{S_m T_m}{L_c} \qquad a = \frac{Mf}{DdL_c} = \frac{s_m T_m}{L_c} \qquad b = \frac{Md}{mD} = \frac{T_m}{t_m}$$

These parameters can be interpreted as follows: A and a are the capture lengths traversable at maximum speed within T_m by the predator and prey, respectively, (corresponding roughly to their agilities) and b gives how many times longer the predator takes than the prey to reach its maximum speed in the absence of drag (corresponding roughly to the relative maneuverability of the prey compared to the predator).

In general, we examine the case where the prey is less agile but more maneuverable then the predator (i.e. $A > a$, $b > 1$) to see how the predator can best take advantage of greater maximum speed and overcome its maneuverability hurdle.

Naive Strategies

The most naive strategy for the predator would be to simply propel itself towards the prey, giving

$$\hat{\mathbf{R}}_n = \frac{\mathbf{y} - \mathbf{Y}}{\|\mathbf{y} - \mathbf{Y}\|}$$

A real-world motivated prey strategy is to pursue the same direction with some fixed angle offset α, with 45° being a typical value for deer pursuit (Stankowich et al.); this yields

$$\theta_n = \mathrm{atan2}(\hat{\mathbf{R}}_n.y, \hat{\mathbf{R}}_n.x) \qquad \hat{\mathbf{r}} = (\cos(\theta_n + \alpha), \sin(\theta_n + \alpha))$$

Even for the simplest of strategies, analytic solution of the steady state can be intractable, but numerical solutions are still illustrative. Figure 1 shows the necessity of the prey's angle offset when $A > a$; in the second case's steady state, the prey orbits a concentric circle within the predator's own circular orbit.

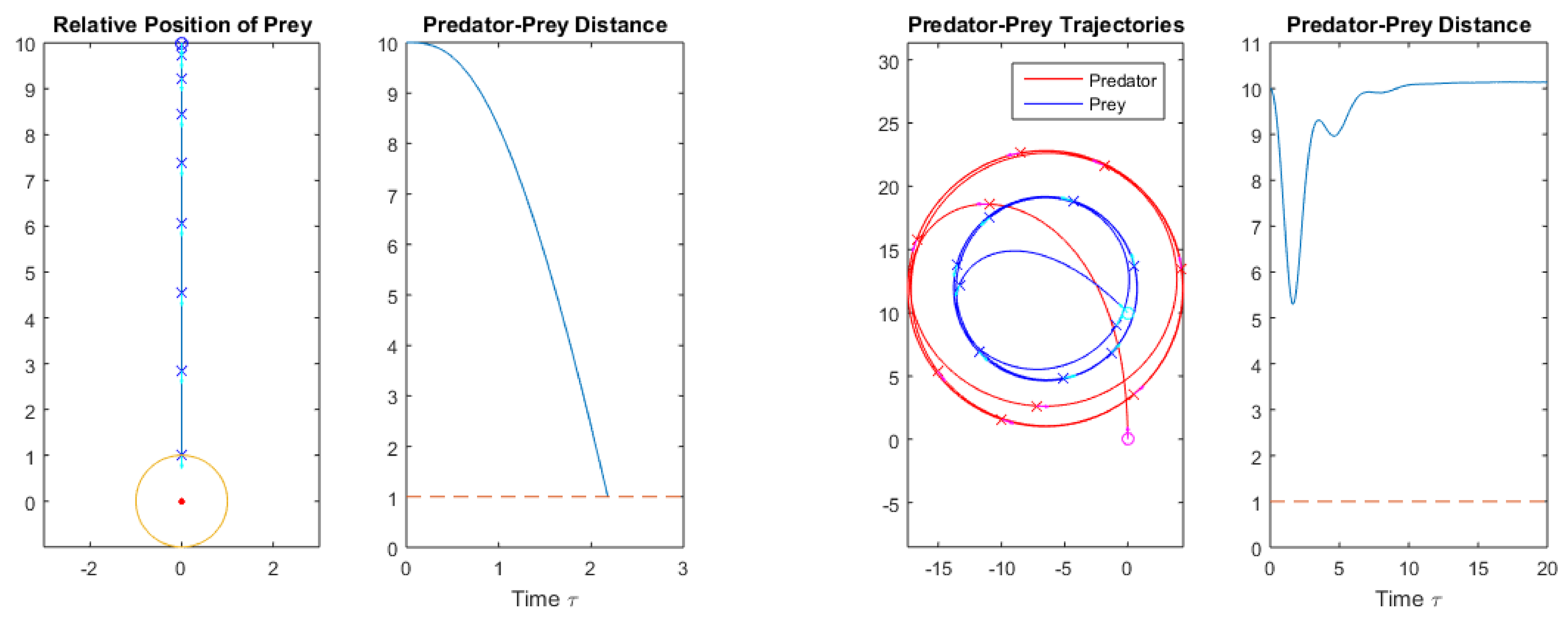

Figure 1: Naive strategies with $A = 20$, $a = 10$, $b = 2$, both animals initially at rest, and the prey initially 10 units ahead of the predator. On the left, $\alpha = 0°$. On the right, $\alpha = 45°$. The orange circle and dashed line represent the capture radius.

Further Predator Strategies

Prey Interception

Suppose the predator takes the velocity of the prey into account. If it assumes that the prey's velocity will be constant, then the predator's ideal velocity might be its maximum speed A directed at some direction $\hat{\mathbf{R}}_i$ so as to intercept the prey at some time τ_i:

$$\mathbf{Y} + A\hat{\mathbf{R}}_i\tau_i = \mathbf{y} + \mathbf{y}'\tau_i$$

Letting $\mathbf{y}_{rel} = \mathbf{y} - \mathbf{Y}$ and solving for τ_i yields a quadratic equation with the positive solution

$$\tau_i = \frac{1}{A^2 - \|\mathbf{y}'\|^2}\left(\mathbf{y}' \cdot \mathbf{y}_{rel} + \sqrt{\left(\mathbf{y}' \cdot \mathbf{y}_{rel}\right)^2 + \left(A^2 - \|\mathbf{y}'\|^2\right)\|\mathbf{y}_{rel}\|^2}\right) \qquad \hat{\mathbf{R}}_i = \frac{1}{A}\left(\mathbf{y}' + \frac{\mathbf{y}_{rel}}{\tau_i}\right)$$

Smooth and Aggressive Slowdown

Suppose the predator takes its own velocity into account. If the predator has some desired velocity $\mathbf{Y}'_d$ and wants to smoothly accelerate from its current velocity to $\mathbf{Y}'_d$, then it might desire to self-propel in a direction $\hat{\mathbf{R}}_s$ such that $\mathbf{Y}'' = A\hat{\mathbf{R}}_s - \mathbf{Y}'$ has the same direction as $\mathbf{Y}'_d - \mathbf{Y}'$; in this case, $\mathbf{R}_s = \mathbf{Y}'_d$. Suppose instead that, to avoid situations like the second trajectory of Figure 1 where orbiting with too much momentum prevents it from reaching the prey, the predator further desires to focus on reducing its current momentum by some 'speed cutting' constant c as follows:

$$\hat{\mathbf{R}}_a = \frac{A\hat{\mathbf{v}} - c\mathbf{Y}'}{\|A\hat{\mathbf{v}} - c\mathbf{Y}'\|}, \text{ where } \hat{\mathbf{v}} = \frac{\mathbf{Y}'_d}{\|\mathbf{Y}'_d\|}$$

Note that if the desired velocity is given by $\mathbf{Y}'_d = A\hat{\mathbf{R}}_n$, i.e. maximum speed towards the prey, then $c = 0$ corresponds to the naive strategy, while $c = 1$ corresponds to self-propelling in direction of $A\hat{\mathbf{R}}_n - \mathbf{Y}'$.

Trials and Case Discussions

The predator strategies surveyed are plain naive $\hat{\mathbf{R}}_n$, naive with smooth slowdown $\hat{\mathbf{R}}_s$ ($\mathbf{Y}'_d = A\hat{\mathbf{R}}_n$ and $c = 1$), naive with aggressive slowdown $\hat{\mathbf{R}}_a$ ($\mathbf{Y}'_d = A\hat{\mathbf{R}}_n$ and $c = 2$), plain prey interception $\hat{\mathbf{R}}_i$, and prey interception with smooth slowdown $\hat{\mathbf{R}}_{is}$ ($\mathbf{Y}'_d = A\hat{\mathbf{R}}_i$ and $c = 1$).

	Situation Parameters					Capture Time / Steady-State Distance				
Case	A	a	b	α	$\|\mathbf{y}_{rel}(0)\|$	$\hat{\mathbf{R}}_n$	$\hat{\mathbf{R}}_s$	$\hat{\mathbf{R}}_a$	$\hat{\mathbf{R}}_i$	$\hat{\mathbf{R}}_{is}$
1	20	10	2	45°	10	10.14	3.03	5.19	8.20	**1.99**
2	20	10	2	15°	10	6.31	2.22	41.92	8.69	**2.15**
3	20	10	2	45°	100	10.14	3.03	43.27	**9.71**	9.80
4	20	10	2	15°	100	6.31	**11.14**	387.69	11.19	11.21
5	20	10	10	45°	10	12.72	6.41	4.80	7.99	**2.22**
6	20	10	10	15°	10	11.38	-	46.93	9.82	**2.60**
7	30	10	1.1	45°	10	2.77	2.19	1.20	3.88	**1.11**
8	30	10	1.1	15°	10	1.17	1.15	1.15	1.15	**1.14**
9	11	10	10	45°	10	17.15	8.55	∞	7.7	**12.38**
10	11	10	10	15°	10	15.96	5.12	∞	9.76	**18.47**

Table 1: Trajectory results for the 5 strategies in 10 situations. Cases 1 is the default situation explored before. Case 3 and case 5 drastically raise initial distance and relative prey maneuverability, respectively. Case 7 represents an ideal situation for the predator with $A \gg a$ and $b \approx 1$, while case 9 represents a difficult situation for the predator with $A \approx a$ and $b \gg 1$. Cases 2, 4, 6, 8, and 10 are identical to the preceding cases with smaller prey angle offset. The most successful strategy for each case is bolded. Strategy $\hat{\mathbf{R}}_s$ for case 6 reaches no constant steady-state distance (plotted below).

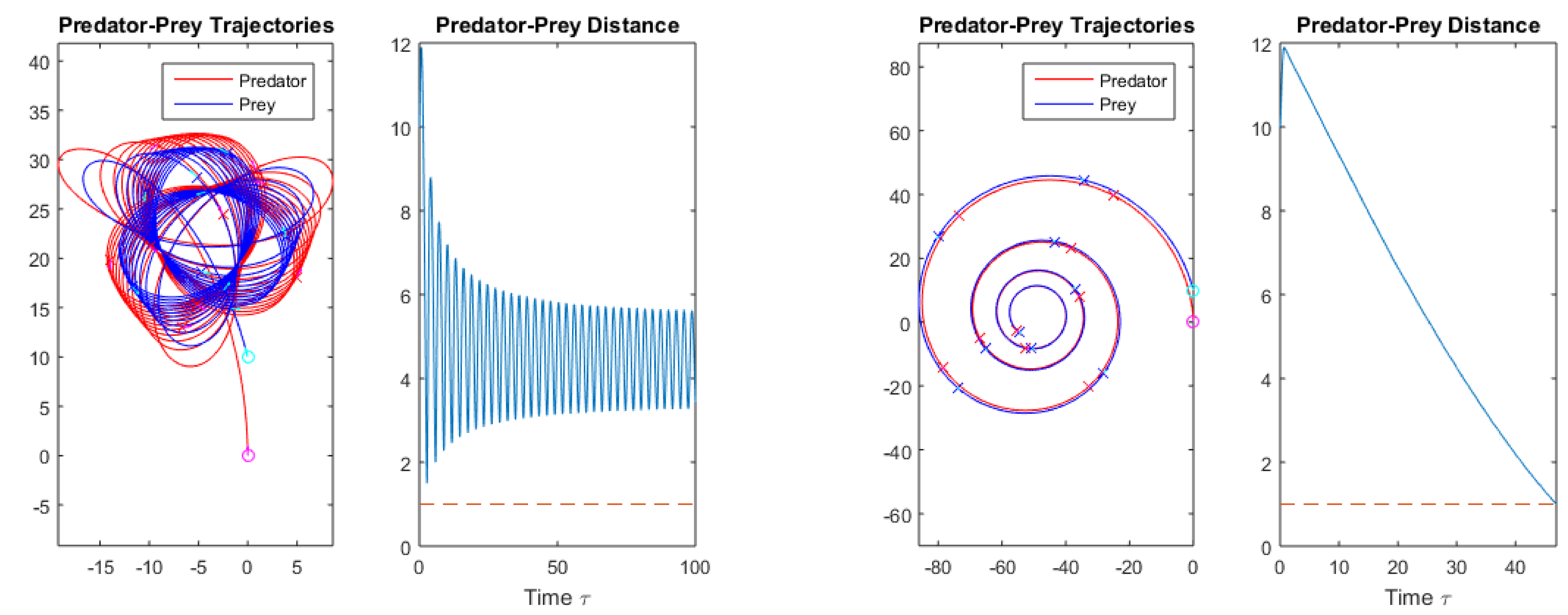

Figure 2: Case 6. Strategy $\hat{\mathbf{R}}_s$ (left) exhibits a complex orbit with a steady-state distance that oscillates at constant magnitude. Strategy $\hat{\mathbf{R}}_a$ (right) is able to capture the prey by reducing speed to the prey's level and exploiting the prey's offset angle to produce a converging spiral trajectory. In situations where predator agility is significantly greater than prey agility, strategy $\hat{\mathbf{R}}_a$ usually results in (often lengthy) capture, but otherwise the slowdown can result in divergence.

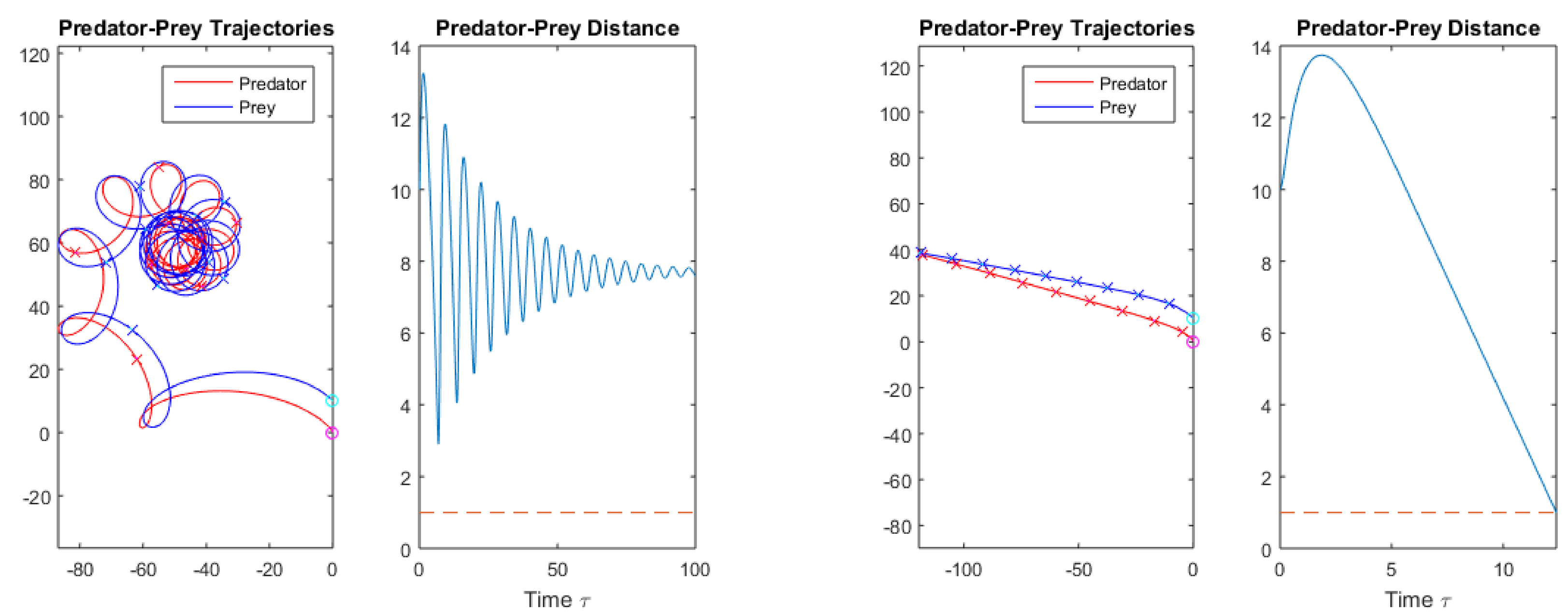

Figure 3: Case 9. Strategy $\hat{\mathbf{R}}_i$ (left) exhibits a series of loops that converge to a typical concentric circular orbit. Strategy $\hat{\mathbf{R}}_{is}$ (right) is able to capture the prey since the slowdown modification causes its trajectory to go wider left towards the prey's direction of motion, reducing the curvature in the prey's trajectory. Across the surveyed cases, the combination of prey interception and smooth slowdown is the most robust strategy and typically produces the best result.

References

Theodore Stankowich, Richard G. Coss; Effects of risk assessment, predator behavior, and habitat on escape behavior in Columbian black-tailed deer. Behav Ecol 2007; 18 (2): 358-367.