Worksheet #6: Asymptotic analysis

Part A

(1) Is $\tan\epsilon = o(\epsilon)$ as $\epsilon \to 0$?

$$\lim_{\varepsilon\to 0} \frac{\tan\varepsilon}{\varepsilon} = \lim_{\varepsilon\to 0} \frac{\sin\varepsilon}{\varepsilon\cos\varepsilon} \overset{LH}{=} \lim_{\varepsilon\to 0} \frac{\cos\varepsilon}{(-\varepsilon\sin\varepsilon + \cos\varepsilon)} = \frac{1}{1} \neq 0.$$

$$\Rightarrow \tan\varepsilon \neq o(\varepsilon)$$

(2) Is $\tan\epsilon = O(\epsilon)$ as $\epsilon \to 0$?

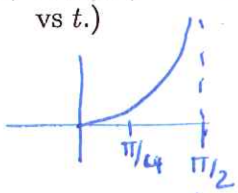
we know the limit is 1 $\Rightarrow$ we can choose any $M > 1$ s.t. $\forall \varepsilon \in [0, c)$ $\frac{\tan\varepsilon}{\varepsilon} < M$

$$\Rightarrow \tan\varepsilon = O(\varepsilon)$$

Part B

Let $f(t,\epsilon) = \epsilon \tan t$.

(1) Is $f(t,\epsilon)$ uniformly convergent to zero on the interval $(0, \pi/4)$ as $\epsilon \to 0$? (Hint: graph f vs t.)



Yes. $f(t,\varepsilon) \leq \varepsilon \tan(\pi/4)$ $\forall t \in [0, \pi/4)$

and $\lim_{\varepsilon\to 0} \varepsilon \tan(\pi/4) = 0$.

(2) Is $f(t,\epsilon)$ uniformly convergent to zero on the interval $(0, \pi/2)$ as $\epsilon \to 0$?

No. since the function is unbounded on this interval.

(3) Does $f(t,\epsilon)$ converge pointwise to zero on the interval $(0, \pi/2)$?

Yes fix $t \in (0, \pi/2)$

Then $\lim_{\varepsilon\to 0} \varepsilon \tan t = \tan t \lim_{\varepsilon\to 0} \varepsilon = 0$.

Part C

(1) Rearrange the terms of the series to form an *asymptotic series* as $\epsilon \to 0$.

$$y(t) = \epsilon^{1/2} y_0(t) + \frac{1}{\epsilon} y_1(t) + \ln(\epsilon) y_2(t) + y_3(t) + \epsilon^2 \ln(\epsilon) y_4(t) + \epsilon^2 y_5(t) + \epsilon \ln^2(\epsilon) y_6(t)$$

Ans: $y(t) = \frac{1}{\varepsilon} y_1(t) + \ln(\varepsilon) y_2(t) + \underset{y_3(t)}{\uparrow} \varepsilon^{1/2} y_0(t) + \varepsilon \ln^2(\varepsilon) y_6(t) + \varepsilon^2 \ln(\varepsilon) y_4(t) + \varepsilon^2 y_5(t)$

or: $\frac{1}{\varepsilon}, \ln\varepsilon, 1, \varepsilon^{1/2}, \varepsilon\ln^2(\varepsilon), \varepsilon^2\ln(\varepsilon), \varepsilon^2$