Worksheet #5: Regular perturbation

Consider the first order differential equation

$$\begin{cases} y' &= -y + \epsilon y^2 \\ y(0) &= 1. \end{cases}$$

(a) Plug $y(t) = y_0(t) + \epsilon y_1(t) + \epsilon^2 y_2(t) + \dots$ into the ODE.

$y_0' + \varepsilon y_1' + \varepsilon^2 y_2' + \dots = -(y_0 + \varepsilon y_1 + \varepsilon^2 y_2 + \dots) + \varepsilon (y_0 + \varepsilon y_1 + \varepsilon^2 y_2 + \dots)^2$

$y_0(0) + \varepsilon y_1(0) + \varepsilon^2 y_2(0) + \dots = 1$

(b) Collect the ϵ^0 terms. What initial condition does y_0 satisfy?

$\begin{cases} y_0' = -y_0 \\ y_0(0) = 1 \end{cases} \Rightarrow \quad y_0 = e^{-t}$

(c) Collect the ϵ^1 terms. What initial condition does y_1 satisfy? [Hint: plug series into original initial condition.]

$\begin{cases} y_1' = -y_1 + y_0^2 = -y_1 + e^{-2t} \\ y_1(0) = 0 \end{cases}$

$y_1' + y_1 = e^{-2t} \qquad \mu = e^t$

$(e^t y_1)' = e^{-t}$

$e^t y_1 = e^{-t} + C \rightarrow y_1 = e^{-2t} + Ce^{-t}$

$y_1(0) = 1 + C = 0 \rightarrow C = -1$

$\Rightarrow y_1(t) = e^{-2t} - e^{-t}$