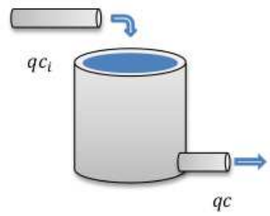
Worksheet #2: Scaling

Consider a chemical reactor tank with flow rate q, volume V, incoming concentration of reactant c_i. We stir the tank so concentration inside $c(t)$ is uniform, so (chemical) mass inside is $Vc(t)$. While inside the tank, the reactant decays at a rate k. In other words, the rate of loss of mass is $kVc(t)$.



a) Write an ODE expressing mass balance:

$$\frac{d}{dt}(Vc(t)) = \underbrace{\qquad\qquad\qquad\qquad}_{\text{mass arrival rate}} - \underbrace{\qquad\qquad\qquad\qquad}_{\text{loss rate}}$$

Now, rewrite this as an ODE for $c'(t)$ and include any relevant initial conditions.

b) Rewrite this ODE using general non-dimensionalization. $\bar{t} = \frac{t}{t_c}$ and $\bar{c} = \frac{c}{c_c}$.

c) Choose $t_c = k^{-1}$, $c_c = c_i$ rewrite the ODE for these choices of characteristic scale using the dimensionless parameters $\gamma := \frac{c_i}{c_0}$ and $\beta := \frac{kV}{q}$.

d) Find another timescale based on the parameters from the original problem. Repeat c) using this time scale and $c_c = c_i$.

e) If we are in a regime where β is very small, which of the choices of timescale give an appropriate reformulation of the problem?